Fall 2026, Math 296: Week 11 Problem Set
Due: Thursday, November 12th, 2026
More Functions

Discussion problems. The problems below should be worked on in class.

(D1) Compare tables in the preliminary problems, and come to a consensus within your group.

(D2) Suppose $f : A \to B$. We say $g : B \to A$ is a *left inverse* of f if $g(f(a)) = a$ for all $a \in A$.

(a) Define $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ by

$$f(a) = 2a \qquad \text{and} \qquad g(b) = \begin{cases} \frac{1}{2}b & \text{if } b \text{ is even;} \\ b - 1 & \text{if } b \text{ is odd.} \end{cases}$$

Prove that g is a left inverse of f. Then locate a **different** left inverse of f.

(b) Define $f : \{1, 2\} \to \{1, 2, 3, 4\}$ by $f(a) = a + 1$. How many left inverses does f have?

(c) Prove the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(a) = a^2$ does not have a left inverse.

(d) Suppose $f : A \to B$ is injective. Briefly explain how you could construct a left inverse $g : B \to A$ of f (hint: decide where g should send each element $b \in B$). Where do you use the fact that f is injective?

(e) Prove that if $f : A \to B$ has a left inverse, then f is injective.

(D3) Suppose $f : A \to B$. We say $g : B \to A$ is a *right inverse* of f if $f(g(b)) = b$ for all $b \in B$.

(a) Recall $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$. Define $f : \mathbb{Z} \to \mathbb{N}_0$ and $g : \mathbb{N}_0 \to \mathbb{Z}$ by

$$f(a) = \begin{cases} a & \text{if } a \geq 0; \\ 2 - a & \text{if } a < 0, \end{cases} \qquad \text{and} \qquad g(b) = b.$$

Prove that g is a right inverse of f. Then locate a **different** right inverse of f.

(b) Define $f : \{-1, 0, 1\} \to \{0, 1\}$ by $f(a) = |a|$. How many right inverses does f have?

(c) Prove the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(a) = a^2$ does not have a right inverse.

(d) Suppose $f : A \to B$ is surjective. Briefly explain how you could construct a right inverse $g : B \to A$ of f. Where do you use the fact that f is surjective?

(e) Prove that if $f : A \to B$ has a right inverse, then f is surjective.

(D4) Fix a function $f : A \to B$.

(a) Prove that if f is bijective, then f has a unique left inverse.

(b) Locate an example of a non-bijection f with a unique left inverse.

(c) Prove if f is bijective, then f has a unique right inverse.

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Recall the *floor* function, which maps $\mathbb{R} \to \mathbb{Z}$ and is defined by

$$\lfloor a \rfloor = \max\{b \in \mathbb{Z} : b \leq a\}.$$

(a) Determine the injectivity and surjectivity of the floor function.

(b) Prove that the function $f : [1, \infty) \to (1, \infty)$ given by

$$a \mapsto 2\lfloor a \rfloor + 1 - a$$

is a bijection by locating its inverse function.

(c) Locate a bijection between the intervals $(0, 1]$ and $(0, 1)$.
Hint: use the composition of the function in part (b) with two other bijections.

(H2) Fix a function $f : A \to B$. Define $g : \mathcal{P}(A) \to \mathcal{P}(B)$ by $C \mapsto f(C)$ and $h : \mathcal{P}(B) \to \mathcal{P}(A)$ by $C \mapsto f^{-1}(C)$.

(a) Suppose $A = \{1, 2\}$, $B = \{3, 4, 5\}$, $f(1) = 5$ and $f(2) = 4$. Find where g and h send each element of their respective domains.

(b) Prove f is injective if and only if g is injective.

(c) If f is injective, is h necessarily injective?

(d) If f is surjective, is h necessarily surjective?