Fall 2026, Math 296: Week 12 Problem Set
Due: Thursday, November 19th, 2026
Relations

Discussion problems. The problems below should be worked on in class.

(D1) Compare answers in the preliminary problems, and come to a consensus within your group.

(D2) *Properties of relations.* Consider the following relations on the set $\mathbb{Z}$:

$$R_{=} = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : a = b\}, \qquad R_{x^2} = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : a^2 = b\},$$
$$R_{\leq} = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b\}, \qquad R_{\text{parity}} = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : b - a \text{ is even}\}.$$

(a) We say a relation R on a set A is *reflexive* if for all $a \in A$, $(a,a) \in R$. Determine which of the above relations is reflexive.

(b) A relation R on a set A is *symmetric* if for all $a, b \in A$, if $(a,b) \in R$, then $(b,a) \in R$. Determine which of the above relations is symmetric.

(c) A relation R on a set A is *antisymmetric* if for all $a, b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$. Determine which of the above relations is antisymmetric.

(d) A relation R on a set A is *transitive* if for all $a, b, c \in A$, if $(a,b) \in R$, and $(b,c) \in R$, then $(a,c) \in R$. Determine which of the above relations is transitive.

(D3) *Function inverses are unique.* Fix a bijection $f : A \to B$. Fill in the blanks in the following proof that the inverse of f is unique.

Proof. Suppose $g : B \to A$ and $h : B \to A$ are both inverses of f (i.e., _____ and _____ for all $a \in A$, and _____ and _____ for all $b \in B$). We wish to prove $g = h$ (i.e., _____ for each $b \in B$). Fix $b \in B$. We see $g(b) = h(__(__(b))) = h(b)$, as desired. □

(D4) *Functions encoded as relations.* For a given function $f : A \to B$, consider the relation

$$R_f = \{(a, f(a)) : a \in A\} \subseteq A \times B$$

between A and B.

(a) Let $f : \{-2, -1, 0, 1, 2\} \to \mathbb{N}_0$ with $n \mapsto |n|$. Write R_f in list notation.

(b) Which of the 4 relations from Problem (D2) can arise from a function in this way?

(c) Fix a relation $R \subseteq A \times B$. Under what conditions does $R = R_f$ for some $f : A \to B$? Fill in the blanks in the following **without** mentioning functions.

R arises from a function if and only if _____$a \in A$, _____$b \in B$ such that _____.

(d) Fix $R \subseteq A \times B$, and suppose $R = R_f$ for some function $f : A \to B$. How can we tell from R whether f is surjective? State your condition **without** mentioning functions.

(e) Fix $R \subseteq A \times B$, and suppose $R = R_f$ for some function $f : A \to B$. How can we tell from R whether f is bijective? State your condition **without** mentioning functions.

(f) Suppose $f : A \to B$ is bijection, and let f^{-1} denote its inverse function. Write an expression for $R_{f^{-1}}$ using set-builder notation, *without* writing "-1".

(g) Given a relation $R \subseteq A \times B$, the *inverse relation* is defined as

$$R^{-1} = \{(b,a) : (a,b) \in R\} \subseteq B \times A.$$

Using your answer to part (c) as a guide, determine what conditions on R ensure R^{-1} is a function. How does this relate to your answer to part (e)?

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) For each of the following relations R on $\mathbb{Z}$, determine (i) whether R is reflexive, (ii) whether R is symmetric, (iii) whether R is antisymmetric, and (iv) whether R is transitive.

(a) $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : |a - b| = 1\}$

(b) $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a = kb \text{ for some } k \in \mathbb{Z}\}$

(H2) Fix a set A, and consider the relation $R_{\subseteq}$ on $\mathcal{P}(A)$ given by

$$R_{\subseteq} = \{(B, B') \in \mathcal{P}(A) \times \mathcal{P}(A) : B \subseteq B'\}.$$

Prove that $R_{\subseteq}$ is reflexive, antisymmetric, and transitive, but not symmetric.

(H3) Suppose A is a finite set, and let $n = |A|$.

(a) How many different relations are there on A?

(b) How many different reflexive relations are there on A?

(c) How many different relations on A are both reflexive and symmetric?