PUBLICATION LIST: CHRISTOPHER O'NEILL

1. Peer-reviewed publications, annotated

(1) *Beyond coins, stamps, and Chicken McNuggets: an invitation to numerical semigroups*,
with Scott Chapman and Rebecca Garcia
to appear, Foundations of Undergraduate Research in Mathematics (Springer).

We give a self contained introduction to numerical semigroups, and present several open problems centered on their factorization properties. (arXiv:1902.05848)

(2) *A computer algebra system for R: Macaulay2 and the m2r package*,
with David Kahle and Jeff Sommars.
To appear, Journal of Statistical Software.

We debut `m2r`, an `R` package that connects `R` to `Macaulay2` through a persistent back-end socket connection running locally or on a cloud server. Topics range from basic use of `m2r` to applications and design philosophy. (arXiv:1706.07797)

(3) *Wilf's conjecture in fixed multiplicity*,
with Winfried Bruns, Pedro García-Sánchez, and Dane Wilbourne
to appear, International Journal of Algebra and Computation.

We give an algorithm to determine whether Wilf's conjecture holds for all numerical semigroups with a given multiplicity m, and use it to prove Wilf's conjecture holds whenever $m \leq 18$. Our algorithm utilizes polyhedral geomtry and methods of lattice point enumeration. (arXiv:1903.04342)

(4) *The geometry and combinatorics of discrete line segment hypergraphs*,
with Deborah Oliveros and Shira Zerbib.
To appear, Discrete Mathematics.

An *r-segment hypergraph* is a hypergraph whose edges consist of r consecutive integer points on line segments in $\mathbb{R}^2$. In this paper, we bound the chromatic number and covering number of hypergraphs in this family, uncovering several interesting geometric properties in the process. (arXiv:1807.04826)

(5) *Elasticity in Apéry sets*,
with Jackson Autry, *Tara Gomes, and Vadim Ponomarenko
to appear, American Mathematical Monthly.

In this note, we ask, given a numerical semigroup, how many elements of its Apéry set have nonunique factorization, and define several new invariants. (arXiv:1908.06448)

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* indicates an undergraduate coauthor

This is an undergraduate research project from the SDSU Mathematics REU in 2019, coadvised with Vadim Ponomarenko.

(6) *Squarefree divisor complexes of certain numerical semigroup elements*,
with *Jackon Autry, *Paige Graves, *Jessie Loucks, Vadim Ponomarenko, and *Samuel Yih.
To appear, Involve, A Journal of Mathematics.

The squarefree divisor complex of a numerical semigroup element $m \in S$ is a simplicial complex Δ_m that arises in the study of multigraded Betti numbers. We compute squarefree divisor complexes for certain classes numerical semigroups, and exhibit a new family of simplicial complexes that occur as the squarefree divisor complex of some numerical semigroup element. (arXiv:1804.06632)

This is an undergraduate research project from the SDSU Mathematics REU in 2017, coadvised with Vadim Ponomarenko.

(7) *Sequentially embeddable graphs*,
with Jackson Autry.
To appear, Journal of Graph Theory.

We call a (not necessarily planar) embedding of a graph G in the plane *sequential* if its vertices lie in $\mathbb{Z}^2$ and the line segments between adjacent vertices contain no interior integer points. In this note, we prove (i) a graph G has a sequential embedding if and only if G is 4-colorable, and (ii) if G is planar, then G has a sequential planar embedding. (arXiv:1812.02904)

(8) *The elasticity of Puiseux monoids*,
with Felix Gotti.
To appear, Journal of Commutative Algebra.

We examine the elasticity invariant for Puiseux monoids (additive submonoids of $\mathbb{Q}_{\geq 0}$). We give a formula, in terms of the atoms, for the elasticity of any Puiseux monoid, and classify when the elasticity is accepted. We also obtain the set of elasticities for a special class of Puiseux monoids, and provide an example of a bifurcus Puiseux monoid (that is, every reducible element has a length 2 factorization). (arXiv:1703.04207)

(9) *A sequence of quasipolynomials arising from random numerical semigroups*,
with *Calvin Leng.
Journal of Integer Sequences **22** (2019), no. 6, Art. 19.6.2.

For a randomly generated numerical semigroup, the expected number of minimum generators can be expressed in terms of a doubly-indexed sequence of integers, denoted $h_{n,i}$, that count generating sets with certain properties. We prove a recurrence that implies the sequence $h_{n,i}$ is eventually quasipolynomial when the second parameter is fixed. (arXiv:1809.09915)

(10) *Augmented Hilbert series of numerical semigroups*,
with *Jeske Glenn, Vadim Ponomarenko, and *Benjamin Sepanski.
Integers **19** (2019), #A32.

Given a numerical semigroup S, the Hilbert series of S (a formal power series equal to the sum of terms t^n over all $n \in S$) can be expressed as a rational function in t whose numerator is characterized in terms of the topology of a simplicial complex determined by membership in S. In this paper, we obtain analogous rational expressions for the related power series whose coefficient of t^n equals $f(n)$ for one of several semigroup-theoretic invariants $f : S \to \mathbb{R}$ known to be eventually quasipolynomial. (arXiv:1806.11148)

This is an undergraduate research project from the SDSU Mathematics REU in 2017, coadvised with Vadim Ponomarenko.

(11) *Factorization length distribution for affine semigroups I: numerical semigroups with three generators*,
with Stephan Ramon García and *Samuel Yih.
European Journal of Combinatorics **78** (2019), 190–204.

Most factorization invariants in the literature extract extremal factorization behavior, such as the maximum and minimum factorization lengths. Invariants of intermediate size, such as the mean, median, and mode factorization lengths are more subtle. We use techniques from analysis and probability to describe the asymptotic behavior of these invariants. Surprisingly, the asymptotic median factorization length is described by a number that is usually irrational. (arXiv:1804.05135)

This is a senior thesis project coadvised with Stephan Ramon García.

(12) *On arithmetical numerical monoids with some generators omitted*,
with *Sung Hyup Lee and *Brandon Van Over.
Semigroup Forum **98** (2019), no. 2, 315–326.

We answer the question "when does omitting generators from an arithmetical numerical monoid S preserve its (well-understood) set of length sets or Frobenius number?" in two extremal cases. (arXiv:1712.06741)

This is an undergraduate research project with UC Berkeley students, advised remotely.

(13) *Random numerical semigroups and a simplicial complex of irreducible semigroups*,
with Jesus De Loera and Dane Wilbourne.
Electronic Journal of Combinatorics **25** (2018), no. 4, #P4.37.

We examine properties of random numerical semigroups under a probabilistic model inspired by the Erdös-Rényi model for random graphs. We provide a threshold function for cofiniteness, and bound the expected embedding dimension, genus, and Frobenius number of semigroups generated with this model. (arXiv:1710.00979)

(14) *Factoring in the Chicken McNugget monoid*,
with Scott Chapman.
Mathematics Magazine **91** (2018), no. 5, 323–336.

We present an accessible introduction to the Chicken McNugget problem: "what numbers of Chicken McNuggets can be ordered using only packs with 6, 9, or 20 pieces?" We also discuss several related questions whose motivation comes from the theory of non-unique factorization. (arXiv:1709.01606)

(15) *On mesoprimary decomposition of monoid congruences*.
Rocky Mountain J. Math. **48** (2018), no. 6, 2069–2085.

We prove two main results concerning mesoprimary decomposition of monoid congruences, which is the first step in a combinatorial method for obtaining primary decompositions of binomial ideals. First, we identify which associated prime congruences appear in every mesoprimary decomposition, thereby completing the theory of mesoprimary decomposition of monoid congruences as a more faithful analog of primary decomposition. Second, we characterize which finite posets can arise as the set of associated prime congruences of a monoid congruence. (arXiv:1708.03441)

(16) *On the computation of factorization invariants for affine semigroups*,
with Pedro García-Sánchez and *Gautam Webb.
Journal of Algebra and its Applications **18** (2019), no. 1, 1950019, 21 pp.

In the setting of affine semigroups, we present the first known algorithm for computing the delta set, an improved algorithm for computing the tame degree, and a dynamic algorithm for computing the catenary degree of semigroup elements. (arXiv:1504.02998)

(17) *Some algebraic aspects of mesoprimary decomposition*,
with Laura Matusevich.
Journal of Pure and Applied Algebra **223** (2018), no. 1, 380–394.

We examine mesoprimary decomposition, a combinatorial method for obtaining primary decompositions of binomial ideals, in the presence of a positive A-grading, where certain pathologies are avoided and the theory becomes more accessible. (arXiv:1706.07496)

(18) *Realizable sets of catenary degrees of numerical monoids*,
with Roberto Pelayo.
Bulletin of the Australian Mathematical Society **97** (2018), no. 2, 240–245

The catenary degree is an invariant that measures the distance between factorizations of elements within an atomic monoid. We classify which finite subsets of $\mathbb{Z}_{\geq 0}$ occur as the set of catenary degrees of a numerical monoid. (arXiv:1705.04276)

(19) *Apéry sets of shifted numerical monoids*,
with Roberto Pelayo.
Advances in Applied Mathematics **97** (2018), 27–35.

We give a highly efficient algorithm for computing Apéry sets of numerical semigroups obtained by "shifting" the minimal generators of a given numerical semigroup, and prove that several numerical monoid invariants, such as the genus and Frobenius number, are eventually quasipolynomial as a function of the shift parameter. (arXiv:1708.09527)

(20) *Minimal presentations of shifted numerical monoids*,
with *Rebecca Conaway, Felix Gotti, *Jesse Horton, Roberto Pelayo, *Mesa Williams, and Brian Wissman.
International Journal of Algebra and Computation **28** (2018), no. 1, 53–68.

We investigate numerical semigroups obtained by "shifting" the minimal generators of a given numerical semigroup. More specifically, we examine minimal relations among the generators when the shift parameter is sufficiently large, culminating in a description that is periodic in the shift parameter. We also explore several applications to computation, combinatorial commutative algebra, and factorization theory. (arXiv:1701.08555)

This is an undergraduate research project from the PURE Mathematics REU in 2015, coadvised with Roberto Pelayo and Brian Wissman.

(21) *Unimodular hierarchical models and their Graver bases*,
with Daniel Bernstein.
Journal of Algebraic Statistics *8* (2017), no. 2, 29–43.

Given a simplicial complex whose vertices are labeled with positive integers, one can associate a vector configuration whose corresponding toric variety is the Zariski closure of a hierarchical model. We classify all vertex-weighted simplicial complexes that give rise to unimodular vector configurations, and provide a combinatorial characterization of their Graver bases. (arXiv:1704.09018)

(22) *On the periodicity of irreducible elements in arithmetical congruence monoids*,
with *Jacob Hartzer.
Integers 17 (2017), #A38.

In this paper, we examine the asymptotic behavior of the set of irreducible elements of arithmetical congruence monoids, and characterize when this set forms an eventually periodic sequence. (arXiv:1606.00376)

This is an undergraduate research project from Texas A&M University.

(23) *On factorization invariants and Hilbert functions.*
Journal of Pure and Applied Algebra **221** (2017), no. 12, 3069–3088.

In the setting of finitely generated semigroups, several factorization invariants, including the delta set, ω-primality, and catenary degree, are expressed in terms of Hilbert functions of multigraded modules. Consequently, several recent results for numerical semigroups are recovered, and each is extended to finitely generated semigroups. (arXiv:1503.08351)

(24) *Mesoprimary decomposition of binomial submodules.*
Journal of Algebra **480** (2017), 59–78.

Mesoprimary decomposition, a combinatorial method for obtaining primary decompositions of binomial ideals, is generalized to "binomial submodules" of certain graded modules over a monoid algebra, analogous to the way primary decomposition of ideals over a Noetherean ring R generalizes to R-modules. (arXiv:1511.00161)

(25) *Sparse solutions of linear Diophantine equations*,
with Iskander Aliev, Jesus De Loera, and Timm Oertel.
SIAM Journal on Applied Algebra and Geometry **1** (2017), no. 1, 239–253.

We present structural results on solutions to the Diophantine system $A\mathbf{y} = \mathbf{b}$, $\mathbf{y} \in \mathbb{Z}^t_{\geq 0}$ with the smallest number of non-zero entries, and discuss some interesting consequences in discrete optimization. Our tools are algebraic and number theoretic in nature and include Siegel's Lemma, generating functions, and commutative algebra. (arXiv:1602.00344)

(26) *Factorization invariants in numerical monoids*,
with Roberto Pelayo.
Contemporary Mathematics **685** (2017), 231-349.

This survey article gives an overview of the length set, elasticity, delta set, ω-primality, and catenary degree invariants in the setting of numerical monoids. For each invariant, we present current major results in the literature and identify the primary open questions that remain. (arXiv:1508.00128)

(27) *On dynamic algorithms for factorization invariants in numerical monoids*,
with *Thomas Barron and Roberto Pelayo.
Mathematics of Computation **86** (2017), 2429–2447.

We investigate the benefits of dynamic programming when computing factorization invariants in numerical monoids. We give dynamic programming algorithms to compute the delta set and ω-primality of any numerical monoid element. (arXiv:1507.07435)

This is an undergraduate research project coadvised with Roberto Pelayo.

(28) *On the set of elasticities in numerical monoids*,
with *Thomas Barron and Roberto Pelayo.
Semigroup Forum **94** (2017), no. 1, 37–50.

We show that the set of length sets for any arithmetical numerical monoid can be completely recovered from its set of elasticities. Additionally, we prove a structure theorem for the set of elasticities of any numerical monoid. (arXiv:1409.3425)

This is an undergraduate research project coadvised with Roberto Pelayo.

(29) *Irreducible decomposition of binomial ideals*,
with Thomas Kahle and Ezra Miller.
Compositio Mathematica **152** (2016), 1319–1332.

We construct irreducible decompositions of binomial ideals by introducing the notion of a soccular monoid congruence; we demonstrate that some binomial ideals do not admit binomial irreducible decompositions. (arXiv:1503.02607)

(30) *On sets of catenary degrees of finitely generated cancellative commutative monoids*,
with Vadim Ponomarenko, *Reuben Tate, and *Gautam Webb.
International Journal of Algebra and Computation **26** (2016), no. 3, 565–576.

We provide structural results on the set of catenary degrees achieved in finitely generated cancellative commutative monoids, including a computable lower bound in terms of the monoid's Betti elements. (arXiv:1506.07587)

This is an undergraduate research project from the SDSU Mathematics REU in 2014, coadvised with Vadim Ponomarenko.

(31) *Numerical semigroups on compound sequences*,
with Vadim Ponomarenko and *Claire Kiers.
Communications in Algebra **44** (2016), no. 9, 3842–3852.

We provide an explicit formula for several factorization invariants of numerical monoids generated by geometric sequences. (arXiv:1503.05993)

This is an undergraduate research project from the SDSU Mathematics REU in 2014, coadvised with Vadim Ponomarenko.

(32) *How do you measure primality?*,
with Roberto Pelayo.
American Mathematical Monthly **122** (2015), no. 2, 121–137.

A self-contained introduction to ω-primality together with a collection of open problems suitable for undergraduates. (arXiv:1405.1714)

(33) *Factorization properties of Leamer monoids*,
with *Jason Haarmann, *Ashlee Kalauli, *Aleesha Moran and Roberto Pelayo.
Semigroup Forum **89** (2014), no. 2, 409–421.

We study the factorization theory of Leamer monoids, and computate several factorization invariants. (arXiv:1309.7477)

This is an undergraduate research project from the PURE Mathematics REU in 2013, coadvised with Roberto Pelayo.

(34) *On the linearity of ω-primality in numerical monoids*,
with Roberto Pelayo.
Journal of Pure and Applied Algebra **218** (2014), no. 9, 1620–1627.

The ω-function measures how far a semigroup element is from being prime. We prove that the ω-function for any numerical semigroup is eventually quasilinear. (arXiv:1309.7476)

(35) *Monoid congruences, binomial ideals, and their decompositions*,
Doctoral dissertation, Duke University (2014).

Extensions of the mesoprimary framework set forth by Kahle and Miller are presented.

2. Submitted preprints, annotated

(1) *Discovery learning in an interdisciplinary course on finite fields and applications*,
with Lily Silverstein.
Submitted.

We describe our approach to teaching a course on finite fields and applications to discrete mathematics using a hybrid of lectures and active learning, with two lecture days and two discovery-learning-based discussions each week. (arXiv:1810.10568)

(2) *Factorization length distribution for affine semigroups II: asymptotic behavior for numerical semigroups with arbitrarily many generators*,
with Stephan Ramon García, Mohamed Omar, and *Samuel Yih.
Submitted.

For numerical semigroups with a specified list of (not necessarily minimal) generators, we obtain explicit asymptotic expressions or quasipolynomial/quasirational representations for all major factorization length statistics. This involves a variety of tools that are not standard in the subject, such as algebraic combinatorics (Schur polynomials and the Jacobi-Trudi formula), probability theory (weak convergence of measures, characteristic functions), and Fourier transforms of distributions. (arXiv:1911.04575)

(3) *On parametrized families of numerical semigroups*,
with *Franklin Kerstetter.
Submitted.

We introduce the notion of weighted factorization lengths, and examine families of numerical semigroups whose generators are parametrized by linear functions. (arXiv:1909.04281)