Fall 2014, Math 302.504
Exam 2 - Review Sheet

Name: ________________________

Given below are some problems to help review for Exam 2. These problems may **not** be turned in for credit, but you are welcome to ask questions about them.

Please note that simply doing these problems will **not** guarantee a passing grade on the exam. You should use these problems as a guide to help determine which topics you feel most comfortable with and which topics you should spend more time reviewing.

(1) Let A, B, C and D be sets.

(a) Show that $(A - B) - C$ is not necessarily equal to $A - (B - C)$.

(b) Prove or disprove: $(A - B) - C = (A - C) - B$.

(c) Prove or disprove: $(A - B) - (C - D) = (A - C) - (B - D)$.

(2) Let $f : \{1, 2, 3, 4\} \to \{a, b, c, d\}$ and $g : \{a, b, c, d\} \to \{1, 2, 3, 4\}$ be functions given by:

$f(1) = d$	$f(2) = c$	$f(3) = a$	$f(4) = b$
$g(a) = 2$	$g(b) = 1$	$g(c) = 3$	$g(d) = 2$

(a) If f one-to-one? Is g one-to-one? Is f onto? Is g onto?

(b) Determine which of the following sums makes sense, and evaluate those that do.

$$\sum_{i=1}^{4} f(i) \qquad\qquad \sum_{\ell \in \{a,b,c,d\}} g(\ell)$$

(3) Determine the value of $\sum_{i=1}^{10000} (i + 1)^2 - (i - 1)^2$.

(4) Find a formula for $\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$ by examining the values of this expression for small values of n. Prove that this formula is correct.

(5) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0, 2^1, 2^2, \ldots$. (Hint: your inductive step will likely have 2 cases.)

(6) Prove that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets with the property that $A_j \subset B_j$ for $j = 1, 2, \ldots, n$, then

$$\bigcup_{j=1}^{n} A_j \subset \bigcup_{j=1}^{n} B_j.$$

(7) Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well-defined, find a formula for $f(n)$ when n is a non-negative integer and prove that your formula is valid.

(a) $f(0) = 1,\ f(n) = -f(n-1)$ for $n \geq 1$.

(b) $f(0) = 1,\ f(1) = 0,\ f(2) = 2,\ f(n) = 2f(n-3)$ for $n \geq 3$.

(c) $f(0) = 0,\ f(1) = 1,\ f(n) = 2f(n+1)$ for $n \geq 2$.