

**Fall 2014, Math 302.504 - Homework Set 1**  
**Due: Wednesday, September 10, 2014**  
**Algorithms and Growth of Functions**

**Name:** \_\_\_\_\_

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Do Problem 2 of Section 3.1 in Rosen.

(2) Do Problem 46 of Section 3.1 in Rosen.

(3) Do Problem 56 of Section 3.1 in Rosen.

(4) Do Problem 8 of Section 3.2 in Rosen. Explain your reasoning for each.

(5) Do Problem 22 of Section 3.2 in Rosen. Explain why your answer is correct.

(6) Determine which of the following functions is  $O(x^2)$ , and give explicit witnesses for each.

(a)  $f(x) = x(\log_2(x))^2$ .

(b)  $f(x) = x(\log_2(x))^3$ .

(c)  $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$ , where  $\lfloor x \rfloor$  and  $\lceil x \rceil$  denote the floor and ceiling functions, respectively.

(d)  $f(x) = \frac{x^2 + 1}{x + 2}$ .

- (7) Given below is pseudocode for a function which takes as input a sorted list  $L$  and an integer  $n \geq 0$ , and returns the index of  $n$  in  $L$ , or  $-1$  if  $n$  does not appear in  $L$ .

**Require:**  $L$ , a sorted list of integers

**Require:**  $n \geq 0$

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function MYSTERYSEARCHMETHOD( $L, n$ )
   $b \leftarrow 0$ 
   $e \leftarrow \text{len}(L) - 1$ 
  while  $b < e$  do
     $m_1 \leftarrow \text{floor}(((e - b)/3) + b)$ 
     $m_2 \leftarrow \text{floor}((2 * (e - b)/3) + b)$ 
    if  $n \leq L[m_1]$  then
       $e \leftarrow m_1$ 
    else if  $n < L[m_2]$  then
       $b \leftarrow m_1 + 1$ 
       $e \leftarrow m_2$ 
    else
       $b \leftarrow m_2 + 1$ 
    end if
  end while
  if  $L[b] == n$  then
    return  $b$ 
  else
    return  $-1$ 
  end if
end function

```

- (a) Explain in words what the function is doing, and compare it to binary search. Is it faster than binary search?
- (b) This function runs in  $O(\log_3(x))$ , where  $x = \text{len}(L)$ . Compare this to binary search, which runs in  $O(\log_2(x))$ . Determine which, if either, is faster, and give a proof (using witnesses).