Fall 2014, Math 302.504 - Homework Set 2
Due: Wednesday, September 17, 2014
Propositional Logic

Name: __________________________

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Which of these are propositions? For those that are propositions, find their truth values.

(a) Do not pass go.

(b) What time is it?

(c) There are no black flies in Maine.

(d) $4 + x = 5$.

(e) The moon is made of green cheese.

(f) $2^n \geq 100$.

(2) Find the negation of each of the following propositions.

(a) Jennifer and Teja are friends.

(b) There are 13 items in a baker's dozen.

(c) Abby sends more than 100 text messages every day.

(d) 121 is a perfect square.

(3) Let P, Q and R be the propositions

P: "You get an A on the final exam."
Q: "You do every exercise in this book."
R: "You get an A in this class."

Write these propositions using P, Q, and R and logical connectives.

(a) You get an A in this class, but you do not do every exercise in this book.

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(c) To get an A in the class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

(4) Determine whether each of these biconditionals is true or false.

(a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.

(b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.

(c) $1 + 1 = 3$ if and only if monkeys can fly.

(d) $0 > 1$ if and only if $2 > 1$.

(5) Write each of these statements in the form "if P, then Q" in English.

(a) If you keep your textbook, it will be a useful reference in your future courses.

(b) To be a citizen of this country, it is sufficient that you were born in the United States.

(c) Speakers will sound their best only if you play them for a long time to break them in.

(d) The Red Wings will win the Stanley Cup if their goalie plays well.

(e) That you get the job implies that you had the best credentials.

(f) The beach erodes whenever there is a storm.

(g) It is necessary to have a valid password to log on to the server.

(h) You will reach the summit unless you begin your climb too late.

(6) Use truth tables to verify the commutative laws

(a) $P \vee Q \equiv Q \vee P$. (b) $P \wedge Q \equiv Q \wedge P$.

(7) Show that each of these conditional statements is a tautology by using truth tables. Do the same without using truth tables.

(a) $[\neg P \wedge (P \vee Q)] \to Q$

(b) $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$

(c) $[P \wedge (P \to Q)] \to Q$

(d) $[(P \vee Q) \wedge (P \to R) \wedge (Q \to R)] \to R$

(8) Show that $\neg(P \oplus Q)$ and $P \leftrightarrow Q$ are logically equivalent using truth tables.

(9) Show that $(P \to Q) \wedge (P \to R)$ and $P \to (Q \wedge R)$ are logically equivalent without using truth tables. What might this property be called?

(10) Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

(a) $Q(0)$

(b) $Q(-1)$

(c) $Q(1)$

(d) $\exists x\, Q(x)$

(e) $\forall x\, Q(x)$

(f) $\exists x\, \neg Q(x)$

(g) $\forall x\, \neg Q(x)$

(11) Translate each of these statements into logical experssions *in three different ways* by varying the domain and by using predicates with one and with two variables.

(a) Someone in your school has visited Uzbekistan.

(b) Everyone in your class has studied calculus and C++.

(c) No one in your school owns both a bicycle and a motorcycle.

(d) There is a person in your school who is not happy.

(e) Everyone in your school was born in the twentieth century.

(12) Determine which of the following universally quantified statements is true, where the domain for all variables consists of all real numbers. For those that are false, give a counterexample.

(a) $\forall x\,(x^2 \neq x)$.

(b) $\exists x\,(x^2 \neq x)$.

(c) $\forall x\,(x^2 \neq 2)$

(d) $\exists x\,(x^2 \neq 2)$

(e) $\forall x\,(|x| > 0)$

(f) $\exists x\,(|x| > 0)$

(13) Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y" and where the domain for x and y consists of all possible systems and all possible states, respectively.

(a) $\exists x\, S(x, \text{open})$.

(b) $\forall x\,(S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$.

(c) $\exists x\, S(x, \text{open}) \vee \exists x\, S(x, \text{diagnostic})$

(d) $\exists x\, \neg S(x, \text{available})$

(e) $\forall x\, \neg S(x, \text{working})$