

Fall 2014, Math 302.504 - Homework Set 3
Due: Wednesday, September 24, 2014
Quantifiers and Rules of Inference

Name: _____

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Let $P(x, y)$ be the statement “Student x has taken class y ”, where the domain for x consists of all students in this class and y consists of all computer science courses at Texas A&M. Express each of the following quantifications in English.

(a) $\exists x \exists y P(x, y)$.

(b) $\forall x \exists y P(x, y)$.

(c) $\exists y \forall x P(x, y)$.

(2) Let $I(x)$ be the statement “ x has an internet connection” and $C(x, y)$ be the statement “ x and y have chatted over the internet,” where the domain for the variables x and y consists of all students in this class. Use quantifiers to express each of the following statements.

(a) Rachel has not chatted over the internet with Chelsea.

(b) Everyone in the class with an internet connection has chatted over the internet with at least one other student in the class.

(c) There are at least two students in the class who have not chatted with the same person.

(d) There are two students in the class who between them have chatted with everyone.

(3) Determine the truth value of each of the following statements if the domain of each variable consists of all real numbers. For each statement, either give a proof or provide a counterexample, whichever is appropriate.

(a) $\forall x \exists y (x^2 = y)$.

(b) $\forall x \exists y (x = y^2)$.

(c) $\exists x \forall y (xy = 0)$.

(d) $\exists x \exists y (x + y \neq y + x)$.

(e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$

(f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$.

(g) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$.

(h) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$.

(4) Find a common domain for the variables x , y , z , and w for which the statement

$$\forall x \forall y \forall z \exists w ((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$$

is true, and another domain for which it is false.

(5) Determine which of the following arguments is valid.

(a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

(b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$. Then $a \neq 0$.

4

(6) Use rules of inference to show that if $\forall x (P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x (P(x) \wedge R(x))$ are true, then $\forall x (R(x) \wedge S(x))$.

(7) Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

(8) Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

- (9) Find a counterexample to the statement that every positive integer can be written as the sum of the squares of three integers.
- (10) (a) Prove that at least one of the real numbers a_1, a_2, \dots, a_n is greater than or equal to the average of these numbers.
- (b) Use part (a) to show that if the first 10 positive integers are placed around a circle, in any order, there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.