

**Fall 2014, Math 302.504 - Homework Set 4**  
**Due: Wednesday, October 1, 2014**  
**Introduction to Proofs**

**Name:** \_\_\_\_\_

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

- (1) (a) Prove or disprove: There exists a rational number  $a$  and an irrational number  $b$  such that  $a^b$  is irrational.

- (b) Prove or disprove: For any rational numbers  $a$  and  $b$ , the number  $a^b$  is rational.

2

(2) Show that if  $r$  is an irrational number, there is a unique integer  $n$  such that the distance between  $r$  and  $n$  is less than  $1/2$ .

(3) Prove that between any two rational numbers there is an irrational number.