Fall 2014, Math 302.504 - Homework Set 5
Due: Wednesday, October 15, 2014
Sets and Functions

Name: _____

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Let A and B be subsets of a set U. Prove that $A \subset B$ if and only if $\overline{B} \subset \overline{A}$.

(2) Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ for each of the following (no proof is required).
   (a) $A_i = \{i, i+1, i+2, \ldots\}$.

   (b) $A_i = \{0, i\}$.

   (c) $A_i = (0, i) = \{x \in \mathbb{R} : 0 < x < i\}$.

   (d) $A_i = (i, \infty) = \{x \in \mathbb{R} : i < x\}$.

(3) Find the domain and range of these functions, and write a formula for each function.

    (a) The function that assigns to each positive integer its last digit.

    (b) The function that assigns to each positive integer its first digit.

    (c) The function that assigns to each positive integer the next largest integer.

    (d) The functions that assigns to each positive integer the largest perfect square not exceeding that integer.

(4) Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is

    (a) one-to-one, but not onto.

    (b) onto, but not one-to-one.

    (c) both one-to-one and onto.

    (d) neither one-to-one nor onto.

(5) Fix $f : A \to B$ and subsets $S, T \subset A$.

    (a) Show that $f(S \cap T) \subset f(S) \cap f(T)$.

    (b) Show that if f is one-to-one, the inclusion in part (a) is an equality.

    (c) Give an example demonstrating that the inclusion in part (a) may be strict.