Fall 2014, Math 302.504 - Homework Set 7
Due: Wednesday, October 29, 2014
Induction, Induction, Induction!

Name: ________________________

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Use induction to prove that a set with $n \geq 2$ elements has $n(n-1)/2$ subsets containing exactly two elements.

(2) Consider this variation of the game Nim. The game begins with single pile of n stones, and players alternate removing stones as in standard Nim. However, the player to take the last stone **loses** (this variation is called *misère play*). Use strong induction to prove that Player 1 has a winning strategy if either $n = 4j$, $n = 4j + 2$ or $n = 4j + 3$ for some integer j, and the second player has a winning strategy if $n = 4j + 1$ for some integer j.

(3) Recall that the n-th Fibonacci number is defines recursively by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$.

(a) Show that

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right)^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$

(b) Use induction to prove that $f_{n+1}f_{n-1} - f_n^2 = (-1)^n$ for all $n \geq 1$.

(c) Give an alternate proof for part (b) by taking determinants in the equation in part (a).