

Fall 2014, Math 302.504 - Homework Set 9
Due: Friday, November 14, 2014
Combinatorics

Name: _____

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) A *bitstring of length n* is a string of n 0's and 1's.

(a) How many bitstrings of length n are possible?

(b) How many bistrings of length n are the same written forward and backward?

(c) How many bitstrings of length n have at least two 1's?

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(2) How many positive integers not exceeding 100 are divisible by either 4 or 6?

(3) Show that there are 2^{2^n} different truth tables for a propositional function with n inputs.

(4) How many different ways are there to arrange the letters a through g so that a and d do not appear next to each other?

(5) Prove that at a party with at least 2 people, there are at least two attendees who know the same number of people there.

(6) Show that if five integers are selected from $\{1, 2, \dots, 8\}$, there must be a pair of selected integers with a sum equal to 9. Is the same true if only 4 integers are selected?

- (7) Bonus question: In the card game Blackjack (slightly simplified), you and the dealer each receive 2 cards out of a standard 52 card deck. Jacks, Queens and Kings are worth 10 points each, Aces are worth 1 point, and every other card (2 through 10) is worth its face value. The goal is for the sum of your cards to total as close to 21 as possible without going over (called *busting*). If you bust, the dealer wins. If you do not bust and the dealer does, you win. If neither you nor the dealer bust, then the player with the higher card sum wins. In case of a tie, the dealer wins.

Under these rules, what is the probability of winning Blackjack against the dealer?