Fall 2014, Math 302.504 - Homework Set 10
Due: Friday, November 21, 2014
Binomial Coefficients

Name: ________________________

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) Give a formula for the coefficient of x^k in the expansion of $((x^2+1)/x)^{100}$.

(2) In this exercise, you will prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ for integers n and k with $1 \leq k \leq n$.

(a) First, prove this identity by arguing that both sides count the number of ways to choose a committee of k people from n people, and a president within the committee.

(b) Second, prove this identity using algebra and the formula for binomial coefficients.

(3) In this exercise, you will use the binomial theorem to give an alternative proof of the following identity:

$$\binom{n+1}{i} = \binom{n}{i} + \binom{n}{i-1}.$$

Use the following steps as a guide.

(a) Use the binomial theorem to expand $(x+y)^{n+1}$. Do *not* simplify.
(b) Next, apply the binomial theorem to the second factor of $(x+y)(x+y)^n$. Distribute.
(c) Use properties of sigma notation to write your answer to part (b) so that it resembles part (a) everywhere except the coefficients.
(d) Compare the coefficients, and conclude the desired identity holds.

(4) Prove the following identity using a combinatorial proof. You will get credit for simply attempting this question, so please do not collaborate or look up a solution elsewhere.

$$\binom{m+n}{r} = \sum_{i=0}^{r} \binom{m}{i} \binom{n}{r-i}.$$