Fall 2014, Math 302.504 - Homework Set 11
Due: Monday, December 1, 2014
Advanced Counting Methods and Recurrence Relations

Name: ____________________________

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet.

(1) How many different strings can be made using all of the letters in $ORONO$?

(2) How many ways are there to pack 8 identical DVDs into 5 indistinguishable boxes so that each box contains at least one DVD?

(3) There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Furthermore, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

(4) Solve each of the following recurrence relations.

(a) $a_n = 4a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 6$.

(b) $a_n = 4a_{n-1} - a_{n-2} - 6a_{n-3}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 3$.

(5) Let $L_n = L_{n-1} + L_{n-2}$, $L_0 = 2$, and $L_1 = 1$.

(a) Solve the recurrence relation L_n.

(b) Show that $L_n = f_{n-1} + f_{n+1}$, where f_n denotes the n-th Fibonacci number.

(6) In this exercise, we will find the general solution to the nonhomogeneous recurrence relation

$$a_n \quad = \quad 8a_{n-1} - 16a_{n-2} + 5n - 3. \quad (*)$$

As in the homogeneous case, this general solution will involve constants α_1 and α_2.

(a) First, find the general solution (call it $a_n^{(h)}$) to the corresponding homogeneous recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ (this should depend on constants α_1 and α_2).

(b) Find values of c and d so that $a_n^{(p)} = cn+d$ satisfies $(*)$ (called a "particular solution").

(c) Using parts (a) and (b), verify (using induction) that $a_n = a_n^{(h)} + a_n^{(p)}$ satisfies $(*)$.

(d) Use the base case values $a_0 = 1$ and $a_1 = 5$ to find α_1 and α_2 in your answer to part (c).