Fall 2014, Math 302.504 - Homework Set 12
Due: Monday, December 8, 2014
Recurrence Relations and Extra Topics

Name: _____

Given below are the required problems for this assignment. Please submit your answers on a printed copy of this sheet. For this assignment, you may omit either Exercise 3 or Exercise 4.

(1) Solve the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2} + 5^n$, $a_0 = 1$, and $a_1 = 1$.

(2) Let $a_n = \sum_{i=0}^{n} i$. Notice that we can (equivalently) define a_n as a recurrence relation $a_n = a_{n-1} + n$ for $n \geq 1$ and $a_0 = 0$. Solve this recurrence relation, and compare the resulting formula to our usual closed form $n(n+1)/2$ for $\sum_{i=0}^{n} i$.

(3) Use generating functions to find the general solution to the recurrence relation

$$a_n = 12a_{n-1} - 35a_{n-2}, \qquad a_0 = 0, a_1 = 2.$$

(4) Prove that the set $\mathbb{Z}^2$ is countable (Hint: use a picture or diagram to illustrate how one might enumerate the elements of this set).