Fall 2015, Math 431: Honors Project Topics

The goal of each project is to learn about a combinatorics topic not discussed in class, and to introduce it to your classmates. Throughout the semester, the following will be expected.

- Choose a topic. Please speak with me before making your decision, to ensure it is an appropriate level and so that we can narrow down a reasonable set of goals. Your should choose a topic (and have it approved) no later than Friday, September 11th.
- Begin reading the agreed-upon background material. Plan to meet with me at least once every two weeks, starting no later than early October, to ensure that you are on track.
- Write (in LaTeX) a 5 page paper aimed at introducing your topic to a fellow honors student. This should contain ample examples and explanations in addition to just theorem statements and proofs. Keep the following deadlines in mind as you proceed.
  - A rough draft of the paper will be due Thursday, November 19th. This draft will be peer reviewed by a fellow student in the weeks that follow.
  - The final paper will be due on the day of our final exam (Friday, December 11th).
- Give a 20 minute presentation introducing the main ideas of your topic. Presentations will take place in class, during the last couple of weeks of the semester. You should keep in mind your target audience and time available when deciding what and how to present.

Your final grade on the project will be determined by the content, quality, and completeness of your final paper and your presentation. However, I reserve the right to deduct points if you fail to meet regularly or are consistently unprepared for meetings.

Given below are several project ideas. I am open to projects not listed here, but please run them by me before making your decision.

(A) *Matroids.* A matroid is a combinatorial structure that generalizes the notion of linear independence in vector spaces, and also arises in undirected graphs, directed graphs (with a different construction), matching sets, hyperplane arrangements, and many other settings. Many constructions from these individual settings (such as graph duals and characteristic polynomials) can be generalized to matroids, and thus simultaneously generalized to all other settings (one combinatorial structure to rule them all?).

Primary source: F. Ardila's video lectures and notes.

`http://math.sfsu.edu/federico/Clase/Matroids/lectures.html`

(B) *Ehrhart polynomials.* Ehrhart's Theorem states that a certain function counting integer points inside of polytopes (polygons in higher dimensions) is a polynomial whose degree equals the ambiant dimension. Ehrhart's theorem is extremely versitile, and can be used to give combinatorial proofs of many results from far outside the realm of discrete geometry.

Primary source: *Computing the continuous discretely*, by M. Beck and S. Robins.

`http://math.sfsu.edu/beck/papers/noprint.pdf`

(C) *Unlabeled structures.* These can be thought of as "combinatorial structures up to isomorphism." Examples include graphs and trees (under graph isomorphism), rooted trees, permutations (isomorphic = same cycle type), and derangements (permutations with no fixed points). Counting unlabeled structures is a natural question, and is often much more complicated than counting the corresponding labeled structures. At work in the background is the theory of combinatorial species, which takes a more high-level (in fact, category-theoretic) approach to enumerative combinatorics.

Primary source: *A walk through combinatorics* (3rd edition), by M. Bóna, Chapter 18.

(D) *Error correcting codes.* These arise frequently in computer science, and are vital to efficient network transmission and computer storage.

Primary source: *A walk through combinatorics* (3rd edition), by M. Bóna, Chapter 17.

Note: This project is best suited for those interested in computational mathematics or computer science, but no background in computer science is required.

(E) *Computational complexity.*

Alan Touring (recently portrayed in a movie by Benedict Cumberbatch) is often thought of as the first computer scientist, but he was actually a mathematician. In his doctoral dissertation, he developed a theoretical model for what is now called a Turing machine, and used it to determine what computers are capable of computing (over 20 years before the first real computer was even invented). Although a true Turing machine is impossible to build (infinite memory is required), simplified models also arise in computer science, such as finite state machines used in studying regular expressions.

Primary source: *A walk through combinatorics* (3rd edition), by M. Bóna, Chapter 20.

Note: This project is best suited for those interested in computational mathematics or computer science, but no background in computer science is required.

(F) *Combinatorial games.* Combinatorial games are two-player games in which (i) all information is available to both players (i.e. no hidden information), (ii) the same moves are available to each player on their turn, and (iii) two games with the same sequence of moves have the same outcome (i.e. no randomness). These conditions are sufficient to ensure that precisely one player has a strategy, that is, a strategy ensuring victory regardless of his/her opponent's play. Many combinatorial games have the form "last player to move wins" (normal play) or "last player to move loses" (misère play). Misère play generally requires a more complicated strategy than normal play, even in games as simple as Nim.

Primary source: *Misere quotients for impartial games*, by T. Blambeck and A. Siegel.

`http://arxiv.org/abs/math/0609825`

(G) *Invariants of non-unique factorization.* The fundamental theorem of arithmetic states that any positive integer can be factored uniquely as a product of prime (irreducible) integers. Non-unique factorization theory aims to classify aother lgebraic settings in which elements fail to admit unique factorization as a product of irreducible elements. Factorization invariants, which are often combinatorial in nature, play a major role in this field, as they provide a concrete measure of the quantity and distribution of factorizations within a given setting.

Primary source: *TBA*, probably selected published articles.

Note: For this project, one semester of abstract algebra is helpful, but not required.

(H) *Monomial ideals.* In a polynomial ring, a monomial ideal is an ideal generated by monomials. Monomial ideals are very combinatorial in nature, allowing many purely algebraic constructions (such as irreducible decompositions, free resolutions, and Krull dimension) to be expressed combinatorially. Conversely, the study of some combinatorial structures (such as undirected graphs and simplicial complexes) can benefit from algebraic machinery by encoding the objects as monomial ideals.

Primary source: *Monomial ideals*, by J. Herzog and T. Hibi, Chapter 1.

Note: This project requires at least one semester of abstract algebra, but preferably two. At the very least, familiarity with ideals and polynomial rings is essential.