Fall 2015, Math 431: Review Problems
Due: Thursday, October 1th, 2015
Exam 1 Review

Exam review problems. As the name suggests, these problems are intended to help you prepare for the upcoming exam.

(ER1) Let $f_0 = 0$, $f_1 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$. Use induction to prove that

$$f_{n+1}f_{n-1} - f_n^2 = (-1)^n.$$

(ER2) Suppose $n \geq 2$. Pick $n+1$ integers from the set $[2n] = \{1, 2, \ldots, 2n\}$. Is it necessarily true that one of the selected integers is twice another? Is it necessarily true that one of the selected integer is a multiple of another?

Note: if you prefer, you may restrict your attention to the case $n = 8$.

(ER3) How many 6-digit positive integers are there whose digits sum to at most 51?

(ER4) A classroom has 25 students, 10 males and 15 females. How many ways are there to form a committee of 5 people, with at least one male and one female?

(ER5) How many 5-digit positive integers are there whose digits sum to a multiple of 5?

(ER6) A *palendrome* is a sequence that reads the same forward and backward. How any ways are there to list the digits in $(1, 1, 2, 2, 3, 3, 4, 4)$ so that the middle 4 digits form a palendrome?

(ER7) Use the binomial theorem to prove that

$$\sum_{\substack{k=0 \\ k \text{ even}}}^{n} \binom{n}{k} 2^k = \frac{3^n + (-1)^n}{2}.$$

(ER8) Prove that

$$\sum_{a_1+a_2+a_3=n} \binom{n}{a_1, a_2, a_3} (-1)^{a_2} = 1.$$

(ER9) Give a combinatorial proof that

$$\binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1}.$$

(ER10) How many 2×2 matrices with entries in $\{0, 1, 2 \ldots, n\}$ have at least one nonzero entry in each row and column?

(ER11) Find a closed formula for $S(n, 2)$.