

Math 16A, Section 3 - Fall 2017
Instructor: Christopher O'Neill
Practice Final Exam

Last Name: _____ **First Name:** _____

Directions:

- The use of a calculator, cell phone, laptop or computer is prohibited.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- Answer all of the questions, and present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but on the quality and correctness of the work leading up to it.

The UC Davis Code of Academic Conduct

I will conduct myself with honesty, fairness, and integrity.

Signature: _____

(1) Fill in the blanks based on the graph of $f(x)$ below.

$$\lim_{x \rightarrow -3} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow 6} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}} \quad f(1) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$$

$f(x)$ has vertical asymptotes at $\underline{\hspace{4cm}}$ (list the locations)

$f(x)$ is decreasing on the interval $(-\infty, \underline{\hspace{2cm}})$ (make the interval as large as possible)

$f(x)$ is increasing on the interval $(\underline{\hspace{2cm}}, 3)$ (make the interval as large as possible)

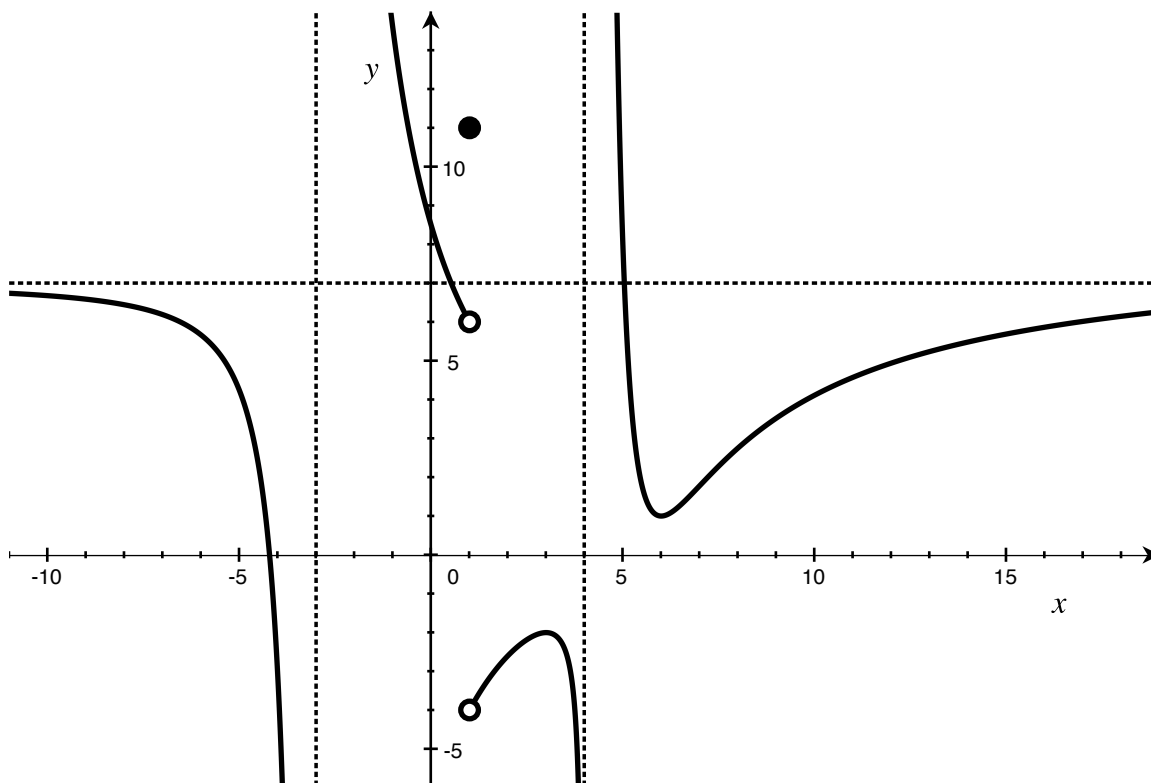
The critical numbers of $f(x)$ are $\underline{\hspace{4cm}}$

$f(x)$ has relative minima at $\underline{\hspace{4cm}}$ (list the locations)

$f(x)$ has relative maxima at $\underline{\hspace{4cm}}$ (list the locations)

At $x = 2$, the graph is concave $\underline{\hspace{1cm}}$, meaning $f''(x) \underline{\hspace{1cm}} 0$ (use “<”, “>”, or “0”)

$f(x)$ changes concavity $\underline{\hspace{1cm}}$ times



(2) Evaluate each of the following limits.

(a) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

(b) $\lim_{x \rightarrow \infty} \frac{4x^3 + 3x^2 + 2x + 1}{2x^3 + 200}$

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$$(c) \lim_{x \rightarrow 1} \frac{x - 1}{x^3 - 2x^2 + x}$$

$$(d) \lim_{x \rightarrow 2} \frac{(x - 3)^2 |x - 2|}{(x - 2)}$$

(3) Find the derivative of each of the following functions.

(a) $f(x) = 4x^4 + 4\sqrt{x} - \frac{1}{x^3}$

(b) $f(x) = (x^5 + x^4 + x^3 + x^2 + x + 1)^7$

(c) $f(x) = \tan(x) \cot(x)$

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$$(d) f(x) = \frac{x^2 + \sin(x)}{1 + \sqrt{x}}$$

$$(e) f(x) = \sqrt{\cos(x)}$$

- (4) Find the equation for the tangent line to the given function at the specified point.

$$f(x) = \frac{x^2 - x - 4}{x^2 + 2}, \quad x = 0$$

- (5) Find $\frac{dy}{dx}$ using implicit differentiation.

$$x^2y^2 - xy = x^2 - 6y$$

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- (6) Suppose a farmer wants to build a fence around a rectangular region. There is 80ft of fence to use, but each side must be at least 10ft long. What is the maximum area the region can have?

- (7) Sketch a graph of the following function. Be sure to identify all zeros, asymptotes, critical numbers, and points of inflection, as well as where the function is increasing, where it is decreasing, where it is concave up, and where it is concave down.

$$f(x) = \frac{3(x^2 - 16)}{(x - 2)^2}$$

Tip: showing your work in an *organized* fashion makes it easier to get partial credit.

Trigonometric Identities

$$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$$

$$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$$

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$$

$$\sin(2A) = 2 \sin(A) \cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\sin^2(A) + \cos^2(A) = 1$$

$$\tan^2(A) + 1 = \sec^2(x)$$

$$1 + \cot^2(A) = \csc^2(x)$$