Math 16A: Short Calculus I
Fall 2017, Section 3
Homework Sheet 3
Due: Wednesday, October 18, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. For each of the following, compute the derivative of $f(x)$ at $x = a$ *using the definition of derivative* (in particular, do not use any "derivative rules" you may or may not have learned in a previous calculus class).

   (a) $f(x) = 2x + 3$, $a = 2$

   $$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{2(2+h)+3-7}{h} = \lim_{h\to 0} \frac{4+2h+3-7}{h}$$
   $$= \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = \boxed{2}$$

   (b) $f(x) = \sqrt{x+3}$, $a = 2$

   $$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\sqrt{h+5}-\sqrt{5}}{h}\left(\frac{\sqrt{h+5}+\sqrt{5}}{\sqrt{h+5}+\sqrt{5}}\right)$$
   $$= \lim_{h\to 0} \frac{h+5-5}{h(\sqrt{h+5}+\sqrt{5})} = \lim_{h\to 0} \frac{h}{h(\sqrt{h+5}+\sqrt{5})} = \lim_{h\to 0} \frac{1}{\sqrt{h+5}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$$

2. Find the equation for the tangent line to $f(x) = x^2 + 3$ at $x = 5$.

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+3-(x^2+3)}{h}$$
   $$= \lim_{h\to 0} \frac{x^2+2xh+h^2+3-x^2-3}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$$
   $$= \lim_{h\to 0} 2x+h = 2x$$

   slope: $f'(5) = 2(5) = 10$
   point: $(5, f(5)) = (5, 28)$

   $$\boxed{y - 28 = 10(x-5)}$$

3. Compute the following limit (note: your answer should have x's' in it, but no h's).

   $$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$
   $$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h} = \lim_{h\to 0} 3x^2+3xh+h^2 = \boxed{3x^2}$$