Math 16A: Short Calculus I
Fall 2017, Section 3
Homework Sheet 7
Due: Monday, November 20, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Consider the following function.

$$f(x) = \frac{x^2 + 3x}{x - 1}$$

Discontinuities: $x-1=0$, $x=1$

(a) For what values of x is $f(x)$ positive? When it $f(x)$ negative?

$f(x)>0$
$x^2+3x=0$
$x(x+3)=0$
$x=0$ or $x=-3$

* $f(x)$ can change pos/neg at $x=0$, $x=-3$, and $x=1$.*

$f(-4)=\frac{4}{-5}$, $f(-1)=\frac{-2}{-2}=1$, $f(0.5)=\frac{+1.75}{-0.5}$, $f(2)=\frac{10}{3}$

positive: $-3<x<0$, $x>1$
negative: $x<-3$, $0<x<1$

(b) Find all critical points of $f(x)$.

$$f'(x)=\frac{(2x+3)(x-1)-(x^2+3x)(1)}{(x-1)^2}$$

$f'(x)=0$: $(2x+3)(x-1)-(x^2+3x)=0$
$2x^2+x-3-(x^2+3x)=0$
$x^2-2x-3=0$
$(x-3)(x+1)=0$

$x=3$ or $x=-1$

(c) For what values of x is $f(x)$ increasing? When is $f(x)$ decreasing?

$f(x)$ can change increasing/decreasing when $x=-1$, $x=1$, and $x=3$.

$f'(-2)=\frac{5}{9}$ $f'(2)=\frac{-3}{1}$
$f'(0)=\frac{-3}{1}$ $f'(4)=\frac{5}{9}$

increasing: $x<-1$, $x>3$
decreasing: $-1<x<1$, $1<x<3$

(d) Which critical points are local maxima? Which are local minima?

$x=-1$ local maximum (increasing, then decreasing)
$x=3$ local minimum (decreasing, then increasing)

(e) For what values of x is $f(x)$ concave up? When is $f(x)$ concave down?

$$f''(x)=\frac{(2x-2)(x-1)^2-(x^2-2x-3)(2x-2)}{(x-1)^4}$$

$f''(0)=\frac{-6}{1}$
$f''(2)=\frac{6}{1}$

$f''(x)=0$: $(2x-2)(x-1)^2-(x^2-2x-3)(2x-2)=0$
$(2x-2)(x^2-2x+1-x^2+2x+3)=0$
$(2x-2)(4)=0$
$x=1$.

concave up: $x>1$
concave down: $x<1$