Math 16A: Short Calculus I
Fall 2017, Section 3
Homework Sheet 9
Due: Friday, December 8, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Graph the following function using the techniques we have seen in class. Identify on your graph all zeros, critical points, points of inflection. Additionally, identify for which values of x the function is positive, where it is negative, where it is increasing, where it is decreasing, where it is concave up, and where it is concave down.

$$f(x) = \frac{x^2 - 9}{x^2 - 4}$$

zeros: $\frac{x^2-9}{x^2-4} = 0$

$x^2 - 9 = 0$

$x = 3, x = -3$

vert. asymptotes:

bottom: $x^2 - 4 = 0$

$x = 2, x = -2$

$\lim_{x \to -2} \frac{x^2-9}{x^2-4}$ "$\frac{-5}{0}$" need two sides

$\lim_{x \to -2^-} \frac{x^2-9}{x^2-4} \Rightarrow \lim_{x \to -2^-} \frac{x^2-9}{(x+2)(x-2)} = -\infty$ (⊖)(⊖)

$\lim_{x \to -2^+} \frac{x^2-9}{(x+2)(x-2)} = \infty$ (⊕)(⊖)

Yes, $x = -2$ vert. asymp

$\lim_{x \to 2} \frac{x^2-9}{x^2-4}$ "$\frac{-5}{0}$" need two sides

$\lim_{x \to 2^-} \frac{x^2-9}{(x-2)(x+2)} = \infty$ (⊖)(⊕)

$\lim_{x \to 2^+} \frac{x^2-9}{(x-2)(x+2)} = -\infty$ (⊕)(⊕)

Yes, $x = 2$ vert asymp.

vertical asymptotes:

$\lim_{x \to \infty} \frac{x^2-9}{x^2-4} = \lim_{x \to \infty} \frac{1 - \frac{9}{x^2}}{1 - \frac{4}{x^2}} = \frac{1}{1} = 1$

$\lim_{x \to -\infty} \frac{x^2-9}{x^2-4} = \lim_{x \to -\infty} \frac{1 - \frac{9}{x^2}}{1 - \frac{4}{x^2}} = \frac{1}{1} = 1$

critical #s:

$f'(x) = \frac{2x(x^2-4) - 2x(x^2-9)}{(x^2-4)^2} = 0$

$2x^3 - 8x - 2x^3 + 18x = 0$

$10x = 0$

$x = 0$

increasing/decreasing:

↓ $(-\infty, -2)$: $f'(-3) = \frac{-30}{+} = -$

↓ $(-2, 0)$: $f'(-1) = \frac{-10}{+} = -$

↑ $(0, 2)$: $f'(1) = \frac{10}{+} = +$

↑ $(2, \infty)$: $f'(3) = \frac{30}{+} = +$

concavity:

$f'(x) = \frac{10x}{(x^2-4)^2}$

$f''(x) = \frac{10(x^2-4)^2 - 10x(2(x^2-4)2x)}{(x^2-4)^2}$

$0 = 10(x^2-4)^2 - 40x^2(x^2-4)$

$0 = (x^2-4)(10x^2 - 40 - 40x^2)$

$0 = \frac{-10}{}(x^2-4)(-3x^2-4)$ always (⊖)

$x = 2, x = -2$

∩ $(-\infty, -2)$: $f''(-3) = \frac{(+)(-)}{(+)} = -$

∪ $(-2, 2)$: $f''(0) = \frac{(-)(-)}{(+)} = +$

∩ $(2, \infty)$: $f''(3) = \frac{(+)(-)}{(+)} = -$

Collect info:

x:		-2		0		2	
	↓		↓	-	↑	:	↑
	∩		∪	∪	∪	:	∩

$f(0) = \frac{9}{4}$