

Fall 2018, Math 320: Week 12 Preliminary Problems
Due: Thursday, November 15th, 2018
More Ideals and Quotient Rings

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Let $R = \mathbb{Z}_{12}$, $I = \langle 4 \rangle$, and $J = \{0, 4, 6\} \subset \mathbb{Z}_{12}$. Notice J is **not** an ideal since $4+6 = 10 \notin J$.

- (a) Using the blanks below, write each of the elements of R/I (i) using bracket notation, (ii) using coset notation, and (iii) as sets.

$$\begin{aligned} \underline{\quad} &= \underline{\quad} = \underline{\quad} \\ \underline{\quad} &= \underline{\quad} = \underline{\quad} \\ [2] &= 2 + I = \{2, 6, 10\} \\ \underline{\quad} &= \underline{\quad} = \underline{\quad} \end{aligned}$$

- (b) Compute the following sum (this should involve adding 9 pairs of integers). Be sure to reduce mod 12! Does the resulting set equal $5 + J$?

$$(2 + J) + (3 + J) = \{2, 6, 8\} + \{3, 7, 9\} = \{ \underline{\quad} \}$$

- (c) Compute the following product (this should involve multiplying 9 pairs of integers). Be sure to reduce mod 12! Does the resulting set equal $6 + J$?

$$(2 + J)(3 + J) = \{2, 6, 8\} \cdot \{3, 7, 9\} = \{ \underline{\quad} \}$$

(P2) Find the kernel of $\varphi : \mathbb{Z}_6 \rightarrow \mathbb{Z}_6$ defined by $\varphi(a) = 2a$.