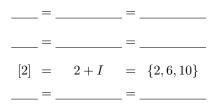
## Fall 2018, Math 320: Week 12 Preliminary Problems Due: Thursday, November 15th, 2018 More Ideals and Quotient Rings

Preliminary problems. These problems should be completed before discussion on Thursday.

- (P1) Let  $R = \mathbb{Z}_{12}$ ,  $I = \langle 4 \rangle$ , and  $J = \{0, 4, 6\} \subset \mathbb{Z}_{12}$ . Notice J is **not** an ideal since  $4+6 = 10 \notin J$ .
  - (a) Using the blanks below, write each of the elements of R/I (i) using bracket notation, (ii) using coset notation, and (iii) as sets.



(b) Compute the following sum (this should involve adding 9 pairs of integers). Be sure to reduce mod 12! Does the resulting set equal 5 + J?

$$(2+J) + (3+J) = \{2, 6, 8\} + \{3, 7, 9\} = \{ \_\_\_ \}$$

(c) Compute the following product (this should involve multiplying 9 pairs of integers). Be sure to reduce mod 12! Does the resulting set equal 6 + J?

$$(2+J)(3+J) = \{2,6,8\} \cdot \{3,7,9\} = \{ \_\_\_ \}$$

(P2) Find the kernel of  $\varphi : \mathbb{Z}_6 \to \mathbb{Z}_6$  defined by  $\varphi(a) = 2a$ .