Fall 2018, Math 320
Midterm Exam Review

The problems below are intended to help you review for the midterm exam, and may *not* be turned in for credit.

(ER1) Is 123456987 prime?

(ER2) Which elements $a \in \mathbb{Z}_9$ have a square root (i.e. an element whose square is a)?

(ER3) Find all elements $a \in \mathbb{Z}_6$ satisfying $a^2 = a$.

(ER4) Determine which elements of $\mathbb{Z}_{10}$ are units, which are zero-divisors, and which are neither.

(ER5) Prove that if $d = (a, b)$ then $d^2 = (a^2, b^2)$.

(ER6) Suppose p is prime. Prove if $a^2 \equiv b^2 \bmod p$, then either $a \equiv b \bmod p$ or $a \equiv -b \bmod p$.

(ER7) Prove or disprove: for all $x, y, z \in \mathbb{Z}$ and $n \geq 2$, if $xz \equiv yz \bmod n$, then $x \equiv y \bmod n$.

(ER8) Consider the set $T = 2\mathbb{Z}$ of even integers, and operations $\oplus$ and $\odot$ given by

$$a \oplus b = a + b \qquad \text{and} \qquad a \odot b = ab/2$$

for all $a, b \in R$. Prove that $(T, \oplus, \odot)$ has associative multiplication, satisfies the distributivity axiom, and that every element of T has an additive inverse.

(ER9) Prove that if R is a commutative ring and

$$S = \{r \in R : ra = 0_R \text{ for all } a \in R\},$$

then S is a subring of R. (Careful: the elements of S are those such that multiplying by **any** element of R yields zero.)

(ER10) Is

$$R = \left\{ \begin{pmatrix} a & a \\ 0 & a \end{pmatrix} : a \in \mathbb{R} \right\} \subset M(\mathbb{R})$$

a subring of $M(\mathbb{R})$?

(ER11) Suppose R is a ring with unity and $a \in R$. Using *only ring axioms*, prove $-a = (-1)a$.

(ER12) Suppose R is a ring and $a, b \in R$. Using *only ring axioms*, prove $-(a+b) = (-a) + (-b)$.

(ER13) Suppose R is a ring and $a \in R$ is **not** a zero-divisor. Prove that if $ab = ac$, then $b = c$.

(ER14) Determine whether the map $\phi : \mathbb{Q} \to \mathbb{Q}$ given by

$$\phi(a) = \begin{cases} \frac{1}{a} & \text{if } a \neq 0; \\ 0 & \text{if } a = 0 \end{cases}$$

is an isomorphism. If not, which of the 4 requirements fail? (It may be more than one!)

(ER15) Let $(T, \oplus, \odot)$ denote the ring from Problem (ER8) above (you may assume T is a ring). Prove $T \cong (\mathbb{Z}, +, \cdot)$.