Fall 2018, Math 320
Final Exam Review

The problems below are intended to help you review for the final exam, and may *not* be turned in for credit.

(ER1) True or false: for all $x, y, z \in \mathbb{Z}$ and $n \geq 2$, if $xz \equiv yz \bmod n$, then $x \equiv y \bmod n$.

(ER2) True or false: if R is a ring and $r, r' \in R$ are units, then $r + r'$ is a unit.

(ER3) Determine whether $f(x) = x^2 + 1$ divides $g(x) = x^4 + 1$ in $\mathbb{Z}_2[x]$. Does the same hold in $\mathbb{Z}_p[x]$ for some/all primes $p > 2$?

(ER4) Is the set

$$R = \{a_d x^d + \cdots + a_1 x + a_0 \mid a_2 = 0\} \subset \mathbb{Q}[x]$$

of polynomials with no terms in degree 2 a ring? If so, is it a field?

(ER5) Determine whether the polynomial $x^3 + 5x^2 + 3x + 4 \in \mathbb{Q}[x]$ is irreducible.

(ER6) Factor $x^4 + 4x^3 + 5x^2 + 2x + 2 \in \mathbb{Z}_7[x]$ as a product of irreducibles.

(ER7) Factor $x^5 + x^4 + x^3 + x^2 + x + 1 \in \mathbb{Z}_2[x]$ as a product of irreducibles.

(ER8) Find all irreducible polynomials of degree 3 in $\mathbb{Z}_2[x]$.

(ER9) Find two distinct monic irreducible polynomials $f(x), g(x) \in \mathbb{Z}_5[x]$ of degree 3. Show the product of $[x + 1]$ and $[x^2 + 2x + 1]$ is different in $\mathbb{Z}_5[x]/\langle f(x)\rangle$ and $\mathbb{Z}_5[x]/\langle g(x)\rangle$.

(ER10) Determine whether $[x + 1]$ is a unit in $\mathbb{Z}_3[x]/\langle x^2 + 1\rangle$.

(ER11) Determine whether $\mathbb{Z}_3[x]/\langle x^3 + 1\rangle$ has zero-divisors.

(ER12) Prove that $G = \{1, 5, 7, 11\} \subset \mathbb{Z}_{12}$ is a group under multiplication (you may state without proof that multiplication in $\mathbb{Z}_{12}$ is associative). Is $G \cong (\mathbb{Z}_2 \times \mathbb{Z}_2, +)$?

(ER13) Determine whether $(\mathbb{Q}_{>0}, +)$ is a group. Determine whether $(\mathbb{Q}_{>0}, \cdot)$ is a group.

(ER14) Multiply the permutations σ and τ given below. Find σ^{-1}. Write σ, τ, and σ^{-1} each as a product of disjoint cycles and as a product of 2-cycles.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 3 & 1 & 4 \end{pmatrix} \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

(ER15) Write all of the elements of A_4 as products of disjoint cycles.