Fall 2019, Math 579: Problem Set 9
Due: Thursday, November 14th, 2019
Recurrence Relations and Generating Functions

Discussion problems. The problems below should be worked on in class.

(D1) *Recurrence relations.* For each of the following, (i) compute $a_0, \ldots, a_5$, (ii) use generating functions to find a formula for a_n in terms of n, and (iii) verify your formula for $n \leq 5$.

(a) $a_0 = 1$, $a_n = 2a_{n-1} + 1$.

(b) $a_0 = 3$, $a_1 = 1$, $a_n = 2a_{n-1} + 3a_{n-2}$.

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Use generating functions to find a_n if $a_0 = 1$ and $a_n = 3a_{n-1} + 2^n$.

(H2) Use generating functions to find L_n if $L_0 = 2$, $L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$.

(H3) Define $c(n) = 1$ if there exists a way to write n as a sum of the values 3 and 5, and $c(n) = 0$ otherwise. For instance, $c(13) = 1$ since $13 = 3 + 5 + 5$, and $c(15) = 1$ since $15 = 5 + 5 + 5$ (as well as $15 = 3 + 3 + 3 + 3 + 3$), but $c(7) = 0$ since there is no way to add 3's and 5's together to obtain 7. Prove that

$$\sum_{n=0}^{\infty} c(n) z^n = \frac{1 - z^{15}}{(1 - z^3)(1 - z^5)}.$$

Conjecture an analogous result if positive integers n_1 and n_2 replace 3 and 5. Note: you are **not** required to prove your conjecture.

Challenge problems. Challenge problems are not required for submission, but bonus points will be awarded for submitting a partial attempt or a complete solution.

(C1) Prove your conjecture from Problem (H3).