Fall 2019, Math 579: Problem Set 11
Due: Tuesday, December 10th, 2019
Exponential Generating Functions

Discussion problems. The problems below should be worked on in class.

(D1) Let ℓ_n denote the number of linear orderings of $[n]$, and let $L(z) = \sum_{n=0}^{\infty} \frac{\ell_n}{n!} z^n$. Give a combinatorial proof that $L(z) = 1 + zL(z)$, and use this to derive the closed form $\ell_n = n!$.

(D2) A *rooted tree* is a tree with a disinguished vertex, called the *root*. Let t_n denote the number of rooted trees with vertex set $[n]$, and let $T(z) = \sum_{n=0}^{\infty} \frac{t_n}{n!} z^n$ denote its exponential generating function. Give a combinatorial proof that $T(z) = ze^{T(z)}$. Hint: what is left if you remove the root of a rooted tree?

(D3) Find the exponential generating function for the number t_n of ways to arrange n books on two bookshelves so that each shelf has at least one book. Then, find a closed form for t_n.

(D4) Suppose from our class of n students, we select an odd number of students to serve on a committee, and select an even number of committee members to serve on a subcommittee. Find the exponential generating function for the number c_n of ways to do this, and then use this to derive a closed formula for c_n.

(D5) Suppose we have n cards. We want to split them into an even number of nonempty subsets, form a line within each subset, then arrange the subsets in a line. Use generating functions to determine the number of different ways to do this.

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Write up the solution to any 2 of the discussion problems.