

**Fall 2019, Math 579: Preliminary Problem Set 12**  
**Due: Thursday, December 5th, 2019**  
**Exponential Generating Functions**

**Preliminary problems.** These problems should be completed before discussion on Thursday.

(P1) A *rooted tree* is a tree with a distinguished vertex, called the *root*. Find all 9 rooted trees with vertex set [3] (it is common to mark the root by circling it).

(P2) Fill in the following table, based on the notes from Tuesday's lecture, to give a combinatorial interpretation of the value of  $c_n$  in terms of  $a_n$  and  $b_n$ .

	Ordinary Generating Functions	Exponential Generating Functions
	$A(z) = \sum_{n=0}^{\infty} a_n z^n, B(z) = \sum_{n=0}^{\infty} b_n z^n,$ $C(z) = \sum_{n=0}^{\infty} c_n z^n$	$A(z) = \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n, B(z) = \sum_{n=0}^{\infty} \frac{b_n}{n!} z^n,$ $C(z) = \sum_{n=0}^{\infty} \frac{c_n}{n!} z^n$
$C(z) = A(z)B(z)$	$c_n = \#$ ways to	$c_n = \#$ ways to
$C(z) = A(B(z))$	$c_n = \#$ ways to	$c_n = \#$ ways to