

**Fall 2019, Math 579**  
**Midterm Exam 1 Review**

The problems below are intended to help you review for the midterm exam, and may *not* be turned in for credit.

(ER1) Let  $f_0 = 0$ ,  $f_1 = 1$ , and  $f_{n+1} = f_n + f_{n-1}$  for  $n \geq 2$ . Use induction to prove that

$$f_{n+1}f_{n-1} - f_n^2 = (-1)^n.$$

(ER2) Suppose  $n \geq 2$ . Pick  $n + 1$  integers from the set  $[2n] = \{1, 2, \dots, 2n\}$ . Is it necessarily true that one of the selected integers is twice another? Is it necessarily true that one of the selected integer is a multiple of another?

Note: if you prefer, you may restrict your attention to the case  $n = 8$ .

(ER3) How many 6-digit positive integers are there whose digits sum to at most 51?

(ER4) A classroom has 25 students, 10 men and 15 women. How many ways are there to form a committee of 5 people with at least one man and one woman?

(ER5) How many 5-digit positive integers are there whose digits sum to a multiple of 5?

(ER6) A *palindrome* is a sequence that reads the same forward and backward. How many ways are there to list the digits in  $(1, 1, 2, 2, 3, 3, 4, 4)$  so that the middle 4 digits form a palindrome?

(ER7) Use the binomial theorem to prove that

$$\sum_{\substack{k=0 \\ k \text{ even}}}^n \binom{n}{k} 2^k = \frac{3^n + (-1)^n}{2}.$$

(ER8) Give a combinatorial proof that

$$\binom{k}{k} + \binom{k+1}{k} + \dots + \binom{n}{k} = \binom{n+1}{k+1}.$$

(ER9) How many  $2 \times 2$  matrices with entries in  $\{0, 1, 2, \dots, n\}$  have at least one nonzero entry in each row and column?

(ER10) Find a closed formula for  $S(n, 2)$ .