

**Fall 2019, Math 579**  
**Midterm Exam 2 Review**

The problems below are intended to help you review for the midterm exam, and may *not* be turned in for credit.

- (ER1) Find all automorphisms of  $K_n$  with one edge removed.
- (ER2) Which complete graphs  $K_n$  have Eulerian cycles? Which complete bipartite graphs  $K_{n,m}$  have Eulerian cycles?
- (ER3) How many Hamiltonian cycles does the wheel graph  $W_n$  have?
- (ER4) Find the number of spanning trees of the complete bipartite graph  $K_{n,2}$  with  $n \geq 2$ . Use the Matrix-Tree Theorem to verify your answer.
- (ER5) Find the chromatic polynomial of the cycle graph  $C_n$ .
- (ER6) Prove that the constant term of the chromatic polynomial of any simple graph  $G$  is 0.
- (ER7) Fix  $n \geq 5$ . What is the maximum number of edges we can remove from  $K_n$  *without* producing a planar graph? What if we require the resulting graph to be connected?  
Challenge: What is the minimum number of edges we can remove from  $K_n$  to produce a planar graph?