Fall 2019, Math 620
Final Exam Review

The problems below are intended to help you review for the midterm exam, and may *not* be turned in for credit.

(ER1) Determine whether there exists an ideal $I \subset \mathbb{Z}[i]$ such that $\mathbb{Z}[i]/I$ is isomorpihc to $\mathbb{Z}_3$.

(ER2) Find the **degree** of the splitting field of $f(x) = x^8 - 1$ over $\mathbb{Q}$.

(ER3) Determine for which q every element of the finite field $\mathbb{F}_q$ has a square root in $\mathbb{F}_q$.

(ER4) Factor $x^5 + 2x^4 + 2x^3 + 2x + 2 \in \mathbb{Z}_3[x]$ as a product of irreducibles.

(ER5) Fix a ring R, an R-module M, and submodules $N, N' \subset M$. Prove that $N \cap N'$ and

$$N + N' = \{n + n' : n \in N, n' \in N'\}$$

are submodules of M.

(ER6) Let $R = \mathbb{Q}[x]$, and consider the R-module $M = R/\langle x^2 \rangle$. Find all submodules of M.

(ER7) Let $R = \mathbb{Q}[x, y]$. Find generators for the kernel and image of the R-module homomorphism $\varphi : R^2 \to R$ given by $(a, b) \mapsto ax + by$.

(Bonus) Determine the initial and terminal objects in the category Slt, if any exist.

(Challenge) Let $R = \mathbb{Q}[x, y]/\langle xy \rangle$, and let $M = R/\langle x \rangle$. Locate free R-modules $F_0, F_1, \ldots$ along with R-module homomorphisms

$$\cdots \to F_2 \to F_1 \to F_0 \to M \to 0$$

so that the kernel of each map is equal to the image of the previous map.