Fall 2021, Math 596: Week 4 Preliminary Problems Due: Thursday, September 16th, 2021 Quasipolynomials and Rational Generating Functions

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Suppose

$$\sum_{n>0} a_n z^n = \frac{Q(z)}{(1-z)^{d+1}}.$$

Fill in the table below.

Conditions on a_n	Conditions on $Q(z)$
a_n is	Q(z) is a polynomial with deg $Q(z) < d + 1$
a_n is a polynomial of degree exactly d	Q(z) is
leading coefficient:	

(P2) Suppose

$$\sum_{n>0} a_n z^n = \frac{R(z)}{(1-z^p)^{d+1}}.$$

Fill in the table below.

Conditions on a_n	Conditions on $R(z)$
a_n is a quasipolynomial	R(z) is
of degree at most d	
and period dividing p	
a_n is a polynomial	R(z) is
of degree at most d	

(P3) We saw in class that if

$$\sum_{n\geq 0} a_n z^n = \frac{1}{(1-z^2)(1-z^3)}$$

then a_n is a quasipolynomial of degree 1 and period 6. Rewrite the right hand side so it matches the theorem at the end of class.