Fall 2021, Math 596: Week 9 Preliminary Problems Due: Thursday, October 21st, 2021 Ehrhart Functions and Ehrhart Series

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Draw the cone

$$C = \operatorname{span}_{\geq 0}\{(1,1), (3,1)\} \subset \mathbb{R}^2.$$

(P2) Using your picture, fill in the blanks in the rational expression for

$$\sigma_C(z_1, z_2) = \sum_{(p_1, p_2) \in C} z_1^{p_1} z_2^{p_2} = \frac{1}{(1 - 1)(1 - 1)}.$$

(P3) Find a formula for f(n) in the series

$$\sigma_C(1,z) = \sum_{n \ge 0} f(n) z^n,$$

and explain its meaning in terms of your picture in Problem (P1).