# Fall 2022, Math 522: Preliminary Problem Set 5 

Due: Wednesday, September 28th, 2022
Modular Arithmetic (Week 2)

Preliminary problems. These problems should be completed before discussion.
(P1) Fix $n \geq 1$, and let $\phi(n)$ denote the number of integers $i \in[1, n-1]$ with $\operatorname{gcd}(i, n)=1$ (this is known as the Euler totient function).
(a) Find $\phi(10)$ and $\phi(12)$.
(b) Let $s=\phi(n)$. A reduced residue system for $n$ is a list of integers $r_{1}, \ldots, r_{s}$ such that

- $\operatorname{gcd}\left(r_{i}, n\right)=1$ for each $i$,
- $r_{i} \not \equiv r_{j} \bmod n$ whenever $i \neq j$, and
- for any $r \in \mathbb{Z}$ with $\operatorname{gcd}(r, n)=1$, we have $r \equiv r_{i} \bmod n$ for some $i$.

Locate 2 distinct reduced residue systems for $n=12$ that share at least one element.

