## Fall 2022, Math 522: Preliminary Problem Set 5 Due: Wednesday, September 28th, 2022 Modular Arithmetic (Week 2)

Preliminary problems. These problems should be completed before discussion.

- (P1) Fix  $n \ge 1$ , and let  $\phi(n)$  denote the number of integers  $i \in [1, n-1]$  with gcd(i, n) = 1 (this is known as the *Euler totient function*).
  - (a) Find  $\phi(10)$  and  $\phi(12)$ .

- (b) Let  $s = \phi(n)$ . A reduced residue system for n is a list of integers  $r_1, \ldots, r_s$  such that
  - $gcd(r_i, n) = 1$  for each i,
  - $r_i \not\equiv r_j \mod n$  whenever  $i \neq j$ , and
  - for any  $r \in \mathbb{Z}$  with gcd(r, n) = 1, we have  $r \equiv r_i \mod n$  for some *i*.

Locate 2 distinct reduced residue systems for n = 12 that share at least one element.