

Fall 2023, Math 320: Preliminary Problem Set 4
Due: Thursday, September 21st, 2023
Introduction To Rings

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Listed below are the axioms for $(R, +, \cdot)$ to be a field. Fill in the blanks.

- For every $a, b, c \in R$,
 - _____
 - _____
 - _____
 - _____
 - _____
 - _____
 - _____
- There exist elements
 - $0 \in R$ (the *additive identity*) such that _____ for all $a \in R$
 - $1 \in R$ (the *multiplicative identity*) such that _____ for all $a \in R$
- For each $a \in R$,
 - there is an element $b \in R$ (the *additive inverse* of a) with $a + b = \underline{\hspace{2cm}} = 0$
 - if $a \neq \underline{\hspace{1cm}}$, there exists $b \in R$ (the *multiplicative inverse* of a) with $ab = ba = \underline{\hspace{2cm}}$

(P2) Write “(I)” next to each item above that is needed to ensure $(F, +)$ is an integral domain.
Note: this requires adding an additional axiom at the end!

(P3) Write “(R)” next to each axiom item that is needed to ensure $(F, +, \cdot)$ is a ring.