Fall 2023, Math 320: Preliminary Problem Set 7
Due: Thursday, October 12th, 2023
Isomorphisms and Homomorphisms

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Suppose R and S are rings, and $\varphi : R \to S$ is any function. Write down the requirements for φ to be an isomorphism.

(P2) Indicate with an (H) which requiremements in (P1) are needed for φ to be a homomorphism.

(P3) Let

$$R_1 = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\} \qquad \text{and} \qquad R_2 = \mathbb{C},$$

and consider the **isomorphism** $\varphi : R_1 \to R_2$ defined by

$$\varphi\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \right) = a + bi$$

for all $a, b \in \mathbb{R}$ (you may assume φ is indeed an isomorphism). Compute

$$\varphi\left(\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ -6 & 5 \end{pmatrix} \right)$$

without doing any matrix multiplication.