Fall 2026, Math 296: Week 3 Problem Set
Due: Thursday, September 17th, 2026
A Brief Excursion Through Propositional Logic

Discussion problems. The problems below should be worked on in class.

(D1) Fix propositions p, q, and r. Verify 5 of the following equivalences using truth tables. Choose **at least 2** equivalences that involve all 3 input propositions!

(a) $\sim(\sim p) \equiv p$ (double negation)
(b) $p \vee q \equiv q \vee p$ (commutativity)
(c) $p \wedge q \equiv q \wedge p$ (commutativity)
(d) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ (associativity)
(e) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ (associativity)
(f) $p \vee p \equiv p \wedge p \equiv p$ (idempotency)
(g) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$ (DeMorgans Law)
(h) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ (DeMorgans Law)
(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (distribution)
(j) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (distribution)

(D2) Fix propositions p, q, r, and s. Verify each of the following equivalences using **only** algebraic properties from Problem (D1) (in particular, do **not** use truth tables). Use only one of the above properties in each step.

(a) $\sim((\sim p) \vee (\sim q)) \equiv p \wedge q$
(b) $r \vee (q \vee (s \vee p)) \equiv ((p \vee q) \vee r) \vee s$

(D3) For given propositions p and q, consider the implication $p \Rightarrow q$:

- its *converse* is the implication $q \Rightarrow p$;
- its *contrapositive* is the implication $\sim q \Rightarrow \sim p$; and
- its *inverse* is the implication $\sim p \Rightarrow \sim q$.

(a) Write the converse, contrapositive, and inverse of the following implication.

"If it is raining, then my windshield wipers are on."

(b) Built a truth table with columns for p, q, and each of the 4 implications above.
(c) Which of these implications are equivalent to one another? Which are not equivalent?
(d) With your truth table as a guide, locate an expression for each implication that involves only or's, and's, and not's. Use the truth table to verify your answer is correct.

(D4) Consider the *nand* operation, defined as $p \uparrow q = \sim(p \wedge q)$ for given propositions p and q.

(a) Demonstrate that the nand operation is commutative but **not** associative.
(b) Use truth tables to demonstrate $\sim p \equiv (p \uparrow p)$ and $p \wedge q \equiv (p \uparrow q) \uparrow (p \uparrow q)$.
(c) Find a compound proposition in p and q that is equivalent to $p \vee q$ that involves only nand operations. Then find nand-only expressions for $p \Rightarrow q$ and $p \Leftrightarrow q$.
(d) Define a new operation "$p \odot q$" within your group by randomly choosing each entry in its truth table definition. Then, find a way to expression $p \odot q$ using only nand's.

(D5) Determine whether each of the equivalences is true or false for all propositions p, q, and r.

(a) $(p \wedge q) \Rightarrow \sim r \equiv \sim r \Rightarrow (p \wedge q)$.
(b) $(p \Rightarrow q) \vee r \equiv \sim p \vee (q \vee r)$

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Write the negation of each of the following propositions. You may want to simplify one or more of these negations before writing them in sentence form, to make the resulting statement clearer to the reader.

(a) $p =$ "Roses are red and violets are blue."

(b) $q =$ "There is exactly one $n \in \mathbb{Z}$ such that $n \cdot m = 0$ for all $m \in \mathbb{Z}$."

(c) $r = p \vee q$.

(H2) Suppose p, q, r, and s are propositions. Simplify

$$\sim((p \wedge q) \vee ((r \vee \sim q) \wedge \sim s))$$

until only basic propositions are negated.

(H3) **Without using truth tables**, prove each of the following equivalences holds for all propositions p, q, and r. You may use the equivalence $(p \Rightarrow q) \equiv (q \vee \sim p)$ you discovered in Problem (D3)(d), as well as any of the equivalences in Problem (D1).

(a) $(q \Rightarrow p) \wedge (q \Rightarrow r) \equiv q \Rightarrow (p \wedge r)$

(b) $\sim((p \Rightarrow q) \wedge (q \Rightarrow p)) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

(H4) *****EXCISED*****

(H5) Determine whether each of the following statements is true or false. Prove your claims.

(a) For all propositions p and q, $(p \Rightarrow q) \wedge (p \Rightarrow \sim q)$ is a tautology.

(b) For all propositions p, q, and r, $\sim(p \vee q) \wedge \sim((\sim q) \vee r)$ is a contradiction.

Challenge problems. Challenge problems are not required for submission, but bonus points will be awarded for submitting a partial attempt or a complete solution.

(C1) In Problem (D4), we saw that any compound proposition is equivalent to one involving only nand's (we say the nand operator is "universal").

(a) Demonstrate that the nor operator is also universal.

(b) Determine whether there are any other universal proposition operators on 2 inputs.