Fall 2026, Math 296: Week 7 Problem Set
Due: Thursday, October 15th, 2026
Sets

Discussion problems. The problems below should be worked on in class.

(D1) Compare answers to the preliminary problems, and come to a consensus within your group.

(D2) Write each of the following sets A in list notation, and find $|A|$.

(a) $A = \{\frac{a}{b} \in \mathbb{Q} : |a|, |b| \leq 4\}$

(b) $A = \{B \subseteq \{1, 2, 3\} : 1 \in B\}$.

(c) $A = \{B \subseteq \{1, 2, 3, 4\} : |B| = 2\}$.

(D3) (a) Locate infinite sets $A, B \subseteq \mathbb{Z}$ for which $A \cup B = \mathbb{Z}$ and $A \cap B = \emptyset$.

(b) Locate sets A, B, C for which $A \cap B \neq \emptyset$, $A \cap C \neq \emptyset$, and $B \cap C \neq \emptyset$, but $A \cap B \cap C = \emptyset$. Hint: this can be done with $|A| = |B| = |C| = 2$.

(c) Suppose $A, B \subseteq \mathbb{Z}$ are finite and that $|A| = 7$, $|B| = 5$, and $|A \cap B| = 3$. Find $|A \cup B|$.

(D4) (a) Fill in the blanks in the following proof that for all sets A and B, $A \setminus B = A \setminus (A \cap B)$.

Proof. Suppose $x \in A \setminus B$. This means _____ and $x \notin B$. Since _____, we must have $x \notin A \cap B$. As such, $x \in A \setminus (A \cap B)$.
Conversely, suppose _____. Then $x \in A$ and _____. Since $x \notin A \cap B$, we have $x \notin A$ ____ $x \notin B$. But $x \in A$, so necessarily $x \notin B$. As such, _____. □

(b) Suppose A and B are subsets of a universal set U. Prove $A \subseteq B$ if and only if $B^c \subseteq A^c$.

(c) Prove or disprove: for any sets A, B, and C, $(A \setminus B) \setminus C = A \setminus (B \setminus C)$.

(D5) Given sets A and B define the *symmetric difference* of A and B as

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

(a) Find $\{1, 2, 3, 4\} \Delta \{3, 4, 5, 6\}$.

(b) Under what conditions is $A \Delta B = A \cup B$? Under what conditions is $A \Delta B = A \cap B$? You do **not** have to prove your assertions.

(c) Suppose A and B are finite sets. Express $A \Delta B$ in terms of the sets $A \cup B$ and $A \cap B$. Prove the formula you find.

(d) Determine whether symmetric difference is commutative.

(e) Determine whether symmetric difference is associative.

(D6) Suppose $A, B, C \subseteq \mathbb{Z}$ satisfy $|A| = |B| = |C| = 7$ and $|A \cap B| = |A \cap C| = |B \cap C| = 5$. What are the possible values of $|A \cap B \cap C|$?

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Locate infinite sets $A, B, C \subseteq \mathbb{Z}$ with $A \cup B \cup C = \mathbb{Z}$ for which $A \cap B$, $A \cap C$, and $B \cap C$ are infinite, but $A \cap B \cap C = \emptyset$.

(H2) Let $A = \{ab : a, b \in \mathbb{Z}, a > 2b\}$. Prove or disprove: $A = \mathbb{Z}$.

(H3) Fix subsets A and B of a universal set U. Consider the sets $C = A^c \setminus B^c$ and $D = (A \setminus B)^c$. Determine whether $C \subseteq D$, and whether $D \subseteq C$.

(H4) Given $n \in \mathbb{Z}$ and $A \subseteq \mathbb{Z}$, define

$$nA = \{na : a \in A\}.$$

(a) Let $A = \{2, 4, 7\}$. Write $5A$ in list notation, and determine whether $8A \subseteq 4\mathbb{Z}$ and whether $6A \subseteq 12\mathbb{Z}$.

(b) Prove $\mathbb{Z} \setminus (2\mathbb{Z} \cup 3\mathbb{Z}) = \{6k + 1 : k \in \mathbb{Z}\} \cup \{6k + 5 : k \in \mathbb{Z}\}$.

(c) Prove or disprove: $18\mathbb{Z} = 2\mathbb{Z} \cap 9\mathbb{Z}$.

(d) Prove or disprove: $18\mathbb{Z} = 3\mathbb{Z} \cap 6\mathbb{Z}$.

(H5) Determine whether each of the following statements is true or false. Prove your claims.

(a) For all sets A, B, C, $(A \cup B) \cap C = A \cup (B \cap C)$.

(b) ****** EXCISED *******