Fall 2026, Math 296: Week 8 Problem Set
Due: Tuesday, October 27th, 2026
More Sets

Discussion problems. The problems below should be worked on in class.

(D1) Compare answers to the preliminary problems, and come to a consensus within your group.

(D2) Let $A = \{1\}$, $B = \mathcal{P}(A)$, and $C = \mathcal{P}(B)$. Write in list notation B, C, and

$$T = \{x \in A \cup B \cup C : \emptyset \in x \text{ or } 1 \in x\}.$$

(D3) Draw each of the following subsets of $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$.

(a) $B = \{1, 2, 4\} \times \mathbb{R}$
(b) $C = \mathbb{Z} \times \{1, 3\}$
(c) $A = \{(x, y) \in \mathbb{R}^2 : (y - x^2)(y - x) = 0\}$
(d) $D = \mathbb{Z} \times \mathbb{Q}$ (this is a hard task; explain why)

(D4) Fix a set A and a collection $P = \{B_i : i \in I\}$ of sets indexed by an index set I. If

- each B_i is nonempty,
- $\bigcup_{i \in I} B_i = A$, and
- $i, j \in I$, $B_i \cap B_j = \emptyset$ for all distinct $i, j \in I$,

then we call P a *partition* of A.

(a) Below are all 5 partitions of $\{1, 2, 3\}$. Find all 15 partitions of $\{1, 2, 3, 4\}$.

$$\{\{1\}, \{2\}, \{3\}\}, \quad \{\{1, 2\}, \{3\}\}, \quad \{\{1\}, \{2, 3\}\}, \quad \{\{1, 3\}, \{2\}\}, \quad \{\{1, 2, 3\}\}$$

(b) A friend says that they "partitioned $\{1, 2, 3\} \times \{4, 5\}$ based on the first coordinate"; what partition is your friend likely talking about?

(c) Partition $\mathbb{Z} \times \mathbb{Z}$ based on the coordinate sum.

(d) Locate a partition of $\mathbb{Z}$ in which no two blocks have the same cardinality.

(D5) Given sets A and B define the *symmetric difference* of A and B as

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

(a) Find $\{1, 2, 3, 4\} \Delta \{3, 4, 5, 6\}$.

(b) Under what conditions is $A \Delta B = A \cup B$? Under what conditions is $A \Delta B = A \cap B$? You do **not** have to prove your claims, but they should be "if and only if" statements.

(c) Express $A \Delta B$ in terms of the sets $A \cup B$ and $A \cap B$. Prove the formula you find.

(D6) Suppose $A, B, C \subseteq \mathbb{Z}$ satisfy $|A| = |B| = |C| = 7$ and $|A \cap B| = |A \cap C| = |B \cap C| = 5$. What are the possible values of $|A \cap B \cap C|$?

Homework problems. You must submit *all* homework problems in order to receive full credit.

(H1) Fix subsets A, B, C, D of a universal set U.

(a) Prove $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$. Demonstrate equality need not hold.

(b) Consider the sets $A^c \times B^c$ and $(A \times B)^c$. What is the natural choice of universal set? Is one of these two sets necessarily contained in the other?

(H2) Recall that $\mathbb{N} = \{1, 2, 3, \ldots\}$, and that for $k \in \mathbb{N}$, we have $k\mathbb{N} = \{kn : n \in \mathbb{N}\}$.

(a) Prove that if $A \subseteq \mathbb{N}$ is finite and nonempty, then $\bigcap_{k \in A} k\mathbb{N}$ is infinite.

(b) Find $\bigcap_{k=1}^{\infty} k\mathbb{N}$.

(H3) Determine whether each of the following statements is true or false. Prove your claims.

(a) Symmetric difference is associative.

(b) There exists an infinite collection of subsets $S_1, S_2, \ldots \subseteq \mathbb{Z}$ such that:

- each S_i is an infinite subset of $\mathbb{Z}$;
- $S_{i+1} \subsetneq S_i$ for each i; and
- $\bigcap_i S_i$ is infinite.

(c) There exists a set A such that $\mathcal{P}(A)$ is a partition of A.

(d) There exists a set A with an element $a \in A$ such that $a \in \mathcal{P}(A)$.