Fall 2026, Math 296: Semester Project

Project Overview

Most of the homework in this course, as well as the problems worked in class, are self-contained proofs. In particular, we are asked to prove a given statement, we prove it, and then we move on. While problems like this are a great way to build and refine our proof-writing skills, it is a bit disingenuous to how mathematics is done in practice: typically, proving "big" (or "interesting") theorems requires writing longer proofs that are too big to tackle all at once. Often, intermediate results (called "lemmas") are used to build towards the larger theorem.

Several considerations must be made when writing and presenting a proof in this scaffolded manner. From the perspective of logic and arguments, one has to consider how to effectively break down a longer proof into bite-sized chunks that (i) have proofs that are easier for the reader to digest, and (ii) have self-contained statements that, as much as possible, stand on their own independent of their context in the larger theorem. Doing so appropriately scaffolds the proof to help the reader follow the overarching argument.

From a writing perspective, a scaffolded approach also necessitates giving the reader some additional description of each intermediate result, to tell them how it fits into the proof of the big result. This often involves including exposition between results to tie them into the overarching story (which, like all mathematical writing, must be clear, precise, and free of ambiguity), as well as concrete, worked-out examples that illustrate the main ideas. These additional ingredients, despite not being essential to check the accuracy of the mathematics, are critical in ensuring the reader sees the big picture as they read your work.

The goal of each potential project is to either work through the proof of a "big" theorem in the manner described above, or to prove a sequence of results that tell a larger story. You will produce a paper that includes your proofs of several results, and contains enough exposition to convey the bigger picture to the reader. Each potential topic covers mathematics not discussed in class, providing an opportunity to explore different areas of mathematics.

Project Selection

Chapter 11 of our textbook contains several potential project topics. Specifically, each section contains a project (and some, like Section 11.5, actually contain several projects). The projects in Sections 11.1-11.5 can be completed without any additional reading, while subsequent sections include projects that may rely on one or more other topics from the book that we have not (yet) covered. Now, this need not stop you from choosing one of the later projects! Just be sure you have looked over the relevant chapters of the textbook before choosing, so your decision is an informed one (of course, I am happy to give you an overview of a particular topic or idea to help in your decision). Any additional background reading necessary for a given project can be found in the book, so no outside sources should be necessary. Additionally, the time/effort needed to learn the additional material will be factored into your overall project goals.

Each project description in Chapter 11, in addition to some background on the topic in question, contains a general outline of the proof(s) and several "exercises" meant to guide you through obtaining and writing the proofs. Your paper should **not** have this structure; instead, your goal should be presenting the theorem and all the details of its proof, alongside examples and exposition, flowing smoothly from one part to the next. The result should be a self-contained document that reads clearly and walks the reader through all of the details of all of the proofs.

Note that some of the project descriptions in Chapter 11 contain more material than is necessary for the project, so be sure to discuss with me before making a final decision. Together, we will set reasonable project goals, so the full scope of the project is clear from its outset. Also, while I am open to projects other than those in our textbook, choosing one will require more up-front effort on your part, to ensure the goals and scope are reasonable.

Project Timeline

Throughout the semester, the following will be expected.

- Choose a topic. Please speak with me before making your decision, to ensure it is an appropriate level and so that we can agree together on a reasonable set of goals. You should choose a topic (and have it approved) no later than **Friday, November 6th**.
- Begin reading the agreed-upon material and working on the proofs. Plan to **meet at least twice** with me throughout the rest of the semester, to ensure that you are on track.
- Write (ideally in LaTeX) a paper aimed at introducing your topic to fellow students, containing ample examples and explanations in addition to the results and proofs you give. Your writing should convey that you understand the intricacies of any proofs presented. Keep the following deadlines in mind as you proceed.
  - A rough draft of the paper will be due **Monday, December 7th** (the start of the last week of class). This will be peer reviewed by a fellow student.
  - The final paper will be due on **Friday, December 18th** (the last day of finals week).
- Your final grade on the project will be determined by the content, quality, and completeness of your final writeup.

Lastly: don't be afraid to ask questions at any point during the project! I do not intend for you to do this project entirely in isolation; rather, it is meant to be a learning experience, and just like our in-class activities and homework assignments, the feedback you get along the way will be a large part of that experience.