MATH 152 - SPRING 2015
ARC LENGTH AND SURFACE AREA OF REVOLUTION FOR PARAMETRIC FUNCTIONS - LECTURE CORRECTIONS

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Problem: Consider the following parametric function.

$$\begin{aligned} x(t) &= t^2 + 1 \\ y(t) &= 2t^3 + t^2 \end{aligned}$$

Find the arc length between $t = 0$ to $t = 2$.

Solution: For a function $y = f(x)$, we would use the following integral.

$$\int_{x=a}^{x=b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

We wish to express this as an integral w.r.t. t. We use the following fact from Math 151

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

to change the integrand, *and* (this is the part I missed in class) we replace dx with $\frac{dx}{dt}dt$. This yields

$$\int_{t=r}^{t=s} \left(\sqrt{1 + \left(\frac{dy/dt}{dx/dt}\right)^2}\right) \frac{dx}{dt}\ dt,$$

which we can also write as

$$\int_{t=r}^{t=s} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

by bringing $\frac{dx}{dt}$ inside the square root.

For the specific functions in this problem, we arrive at

$$\int_{t=0}^{t=2} \sqrt{(2t)^2 + (6t^2 + 2t)^2}\ dt.$$

Note that a similar change must be made for questions involving surface area of revolution for a parametric curve. The resulting integral has the following form (when revolving about the x-axis).

$$\int_{t=r}^{t=s} 2\pi y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$