Spring 2016, Math 485
Week 6 Homework

(Q1) Fix a finite dimensional $\Bbbk$-vector space V and a finite set of vectors $E \subset V$, and let

$$\mathcal{I}(E) = \{I \subset E : I \text{ contains no linear dependencies}\}.$$

Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called *linear*.

(Q2) Fix a finite graph G, and let $E = E(G)$ denote the set of edges, and let

$$\mathcal{I}(G) = \{I \subset E : I \text{ contains no cycles}\}.$$

Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called *graphical*.

(Q3) Fix a bipartite graph $G = (A, E)$, and let

$$\mathcal{I}(E) = \{I \subset E : |N(I)| = |I|\},$$

where $N(I) \subset A$ denote the set of neighbors of vertices in I. Prove that $(E, \mathcal{I}(E))$ forms a matroid. Such matroids are called *transversal*.

(Q4) Prove that every graphical matroid is linear.