Spring 2016, Math 485
Week 9 Homework

(Q1) Prove that matroid dual coincides with graph dual for any graphical matroid whose defining graph is planar.

(Q2) Prove that the dual of any linear matroid is linear.

(Q3) Prove that matroid deletion and contraction are dual operations, that is, $(M/e)^* = M^* \setminus e$ and $(M \setminus e)^* = M^*/e$.

(Q4) Compute $T_G(x, y)$, where G is your favorite graph with at least 4 vertices and 6 edges.

(Q5) Prove for any undirected graph G, $T_G(1, 1)$ equals the number of spanning trees of G.

(Q6) Prove for any undirected graph G, $T_G(2, 1)$ equals the number of acyclic subsets of G.

(Q7) Prove for any undirected graph G, $T_G(1, 2)$ equals the number of spanning subgraphs of G, that is, subgraphs with the same number of vertices and connected components as G.

(Q8) Prove for any undirected graph G, $T_G(2, 0)$ equals the number of acyclic orientation of G, that is, the number of ways to direct the edges of G so that no directed cycles are formed.

(Q9) Prove for any undirected graph G, $T_G(0, 2)$ equals the number of strongly connected orientation of G, that is, the number of ways to direct the edges of G so that there is a directed path between any two vertices.

(Q10) Prove for any undirected graph $G = (V, E)$ with c connected components,

$$(-1)^{|V|-c} k^c T_G(1 - k, 0) = \chi_G(k),$$

where $\chi_G(k)$ equals the number of proper k-colorings of the vertices of G.