Spring 2016, Math 485
Week 10 Homework

(Q1) Prove that every irreducible element in the arithmetic congruence monoid

$$(M_{1,4}, \cdot) = \{n \geq 1 : n \equiv 1 \bmod 4\} \subset (\mathbb{Z}_{\geq 1}, \cdot)$$

is either (i) a prime, or (ii) a product of 2 primes each congruent to 3 mod 4.

(Q2) Prove that any additively closed subset $S \subset (\mathbb{Z}_{\geq 0}, +)$ has a unique minimal generating set $n_1, \ldots, n_k$. In particular, show that if X generates S, then $n_1, \ldots, n_k \in X$, so the remaining elements of X are redundant generators. Hint: let $\{n_1, \ldots, n_k\}$ denote the *irreducible* elements of S.

(Q3) Prove that if a numerical monoid S is minimally generated by $n_1, \ldots, n_k$, then the elasticty $\rho(S)$ is given by $\rho(S) = n_k/n_1$.

(Q4) Suppose S is minimally generated by n_1 and n_2. Characterize the value

$$\min\{\rho(n) : n \in S, \rho(n) \neq 1\}$$

in terms of n_1 and n_2.