Math 16B, Section 1 - Spring 2017
Instructor: Christopher O'Neill
Practice Final Exam

Last Name: ____________________________ **First Name:** ____________________________

Directions:

- The use of a calculator, cell phone, laptop or computer is prohibited.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- Answer all of the questions, and present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but on the quality and correctness of the work leading up to it.

The UC Davis Code of Academic Conduct

I will conduct myself with honesty, fairness, and integrity.

Signature: ____

(1) Find the derivative of each of the following functions.

(a) $f(x) = 3x^2 + 2x + x\sin(x)$

(b) $f(x) = 3e^{2x} + 4x^2e^{5x}$

(c) $f(x) = \ln(x^2 + 2x + 1)$

(2) Solve for x in the following equation.

$$\ln(x-1)+\ln(x-3)=1$$

(3) Suppose you invest \$5000 in a savings account with an interest rate of 5%, compounded monthly.

(a) How much money will be in the account after 5 years?

(b) Suppose that instead of compouding monthly, the interest is compounded continuously. How long will it take for the account balance to reach \$6000?

(4) Suppose a colony of bacteria initially has 1000 bacteria and grows to 1500 bacteria after 5 hours.

(a) Use an exponential growth model to find $P(t)$, the population after t hours.

(b) Based on your model in part (a), what will the population be after 24 hours?

(c) How long does it take the colony to double its population?

(5) Approximate the following integral using trapezoid rule with $n = 4$.

$$\int_0^{\pi} \sin(x)\ dx$$

(6) A skydiver jumps from a plane at 12000ft. Assuming they have not yet deployed their parachute, how fast is the skydiver moving when they reach ground level? Assume the initial velocity of the skydiver is 0 ft/sec, and that acceleration due to gravity is a constant -32 ft/sec^2.

(7) Find the average value of $f(x) = x\sin(x)$ for $0 \leq x \leq \pi/2$.

(8) Evaluate each of the following integrals.

(a) $\int 6x^2 e^{2x}\ dx$

(b) $\int \frac{\cos(\ln(x))}{x}\ dx$

(c) $\int x^2 \sin(x)\ dx$

(d) $\displaystyle\int \frac{x^3+2}{x^2-1}\,dx$

(e) $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin(x)}\ dx$

(f) $\displaystyle\int_0^{\infty} 4xe^{-x^2}\ dx$

(9) Suppose you have a coin with “1” on one side and “3” on the other. Consider flipping this coin 3 times, and let x denote the random variable that records the sum of the values that come up.

(a) List the elements of the sample space.

(b) What is the probability that $x = 3$?

(c) Compute the mean of x.

(d) Compute the variance of x.

(10) Consider the continuous random variable x that takes values in the range $0 \leq x < \infty$ with

$$f(x) = ke^{-2x}$$

as a probability density function.

(a) Find the value for k so that $f(x)$ is a probability density function.

(b) Find $P(1 \leq x \leq 2)$.

(c) Find the median of x.

(d) Find the variance of x.

Trigonometric Identities

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B)\\
\sin(A-B) &= \sin(A)\cos(B)-\cos(A)\sin(B)\\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B)+\sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(2A) &= 2\sin(A)\cos(A)\\
\cos(2A) &= \cos^2(A)-\sin^2(A)
\end{aligned}$$

$$\sin^2(A)+\cos^2(A)=1$$

$$\tan^2(A)+1=\sec^2(x)$$

$$1+\cot^2(A)=\csc^2(x)$$

$$\begin{aligned}
\int \sec(x)\ dx &= \ln|\sec(x)+\tan(x)|+C\\
\int \csc(x)\ dx &= -\ln|\csc(x)+\cot(x)|+C
\end{aligned}$$

Error Estimates

$$|E_T| \leq \frac{M(b-a)^3}{12n^2} \qquad f''(x)\leq M \text{ for all } x\in[a,b]$$