Math 16B: Short Calculus II
Spring 2017, Section 1
Homework Sheet 2
Due: Friday, April 14, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Find the derivatives of the following functions.

(a) $f(x) = \dfrac{\ln(x)}{e^x}$

$$f'(x) = \frac{\left(\frac{1}{x}\right)(e^x) - (\ln(x))(e^x)}{(e^x)^2}$$

(b) $f(x) = \ln(x^2(x+1)^3)$. $= \ln(x^2) + \ln((x+1)^3) = 2\ln(x) + 3\ln(x+1)$

$$f'(x) = \frac{2}{x} + \frac{3}{x+1}$$

2. Using properties of logarithms, write the following using only a single logarithm.

$$3(\ln(x+2) - 4\ln(2x^3) + \ln(x^2+1) + \log_2(x))$$

$= 3\left(\ln(x+2) - \ln((2x^3)^4) + \ln(x^2+1) + \frac{\ln(x)}{\ln(2)}\right)$

$= 3\left(\ln\left(\frac{(x+2)(x^2+1)}{(2x^3)^4}\right) + \ln(x^{1/\ln(2)})\right)$

$= 3\left(\ln\left(\frac{(x+2)(x^2+1)x^{1/\ln(2)}}{(2x^3)^4}\right)\right) = \ln\left(\left(\frac{(x+2)(x^2+1)x^{1/\ln(2)}}{(2x^3)^4}\right)^3\right)$

3. Find the half-life of a radioactive substance for which 99% remains after 1 year.

Model: $R(t) = Ce^{kt}$

99% after 1 yr: $0.99C = Ce^{k \cdot 1}$

$0.99 = e^k$

$k = \ln(0.99)$

$R(t) = Ce^{t\ln(0.99)}$

Half-life: $\frac{C}{2} = Ce^{t\ln(0.99)}$

$\frac{1}{2} = e^{t\ln(0.99)}$

$\ln(\tfrac{1}{2}) = t\ln(0.99)$

$$t = \frac{\ln(\tfrac{1}{2})}{\ln(0.99)}$$