

Math 16B: Short Calculus II
 Spring 2017, Section 1
 Homework Sheet 7
 Due: Wednesday, June 7, 2017

Submit your solutions to the following problems in lecture on the due date above. Present your work in a clean and organized fashion, either on a printed copy of this document (preferred) or a separate sheet of paper. As stated in the syllabus, late submissions will **not** be accepted.

1. Suppose you have a 6-sided die with side labels 1, 2, 2, 4, 4, and 5. Consider the (discrete) random variable x that counts the number of even values that occur when rolling it twice.

$P(11) = 1/36$
 $P(21) = 2/36$
 $P(41) = 2/36$
 $P(51) = 1/36$
 5

(a) Identify all possible outcomes in the sample space, and find the probability of each.

$P(12) = 2/36$ $P(42) = 4/36$ $P(14) = 2/36$ $P(44) = 4/36$ $P(15) = 1/36$ $P(45) = 2/36$
 $P(22) = 4/36$ $P(52) = 2/36$ $P(24) = 4/36$ $P(54) = 2/36$ $P(25) = 2/36$ $P(55) = 1/36$

- (b) Find the expected value (i.e. mean), variance, and standard deviation of x .

$$\mu = (0) \left(\frac{4}{36}\right) + (1) \left(\frac{16}{36}\right) + (2) \left(\frac{16}{36}\right) = \frac{48}{36} = \boxed{\frac{4}{3}}$$

$\underbrace{\hspace{1.5cm}}_{P(X=0)} \quad \underbrace{\hspace{1.5cm}}_{P(X=1)} \quad \underbrace{\hspace{1.5cm}}_{P(X=2)}$

$$V = (0 - \frac{4}{3})^2 P(X=0) + (1 - \frac{4}{3})^2 P(X=1) + (2 - \frac{4}{3})^2 P(X=2)$$

$$= (\frac{16}{9}) \left(\frac{4}{36}\right) + (\frac{1}{9}) \left(\frac{16}{36}\right) + (\frac{4}{9}) \left(\frac{16}{36}\right) = \frac{144}{324} = \boxed{\frac{4}{9}}$$

$$\sigma = \sqrt{V} = \boxed{\frac{2}{3}}$$

2. Let x be a continuous random variable with probability density function

$$f(x) = k \sin(x)$$

for $0 \leq x \leq \pi$.

- (a) Find a value of k so that f is a probability density function.

$$\int_0^{\pi} k \sin(x) dx = -k \cos(x) \Big|_0^{\pi} = (-k(-1)) - (-k(1)) = 2k = 1$$

$$\boxed{k = 1/2 \quad f(x) = \frac{1}{2} \sin(x)}$$

- (b) Find the expected value (i.e. mean), median, variance, and standard deviation of f .

mean: $\mu = \int_0^{\pi} x \left(\frac{1}{2} \sin(x)\right) dx = \boxed{\frac{\pi}{2}}$ (use integration by parts)

variance: $V = \int_0^{\pi} (x - \mu)^2 \left(\frac{1}{2} \sin(x)\right) dx = \int_0^{\pi} \left(x - \frac{\pi}{2}\right)^2 \left(\frac{1}{2} \sin(x)\right) dx = \boxed{\frac{1}{4} \pi^2 - 2}$ (use integration by parts)

standard deviation: $\sigma = \sqrt{V} = \boxed{\sqrt{\frac{1}{4} \pi^2 - 2}}$

Median: find M so that $\int_0^M \frac{1}{2} \sin(x) dx = \frac{1}{2}$ $M = \boxed{\frac{\pi}{2}}$