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PREREQUISITE REVIEW 6.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, factor the expression.

1. $x^2 - 16$

2. $x^2 - 25$

3. $x^2 - x - 12$

4. $x^2 + x - 6$

5. $x^3 - x^2 - 2x$

6. $x^3 - 4x^2 + 4x$

7. $x^3 - 4x^2 + 5x - 2$

8. $x^3 - 5x^2 + 7x - 3$

In Exercises 9–14, rewrite the improper rational expression as the sum of a proper rational expression and a polynomial.

9. $\dfrac{x^2 - 2x + 1}{x - 2}$

10. $\dfrac{2x^2 - 4x + 1}{x - 1}$

11. $\dfrac{x^3 - 3x^2 + 2}{x - 2}$

12. $\dfrac{x^3 + 2x - 1}{x + 1}$

13. $\dfrac{x^3 + 4x^2 + 5x + 2}{x^2 - 1}$

14. $\dfrac{x^3 + 3x^2 - 4}{x^2 - 1}$

EXERCISES 6.3

In Exercises 1–12, write the partial fraction decomposition for the expression.

1. $\dfrac{2(x + 20)}{x^2 - 25}$

2. $\dfrac{3x + 11}{x^2 - 2x - 3}$

3. $\dfrac{8x + 3}{x^2 - 3x}$

4. $\dfrac{10x + 3}{x^2 + x}$

5. $\dfrac{4x - 13}{x^2 - 3x - 10}$

6. $\dfrac{7x + 5}{6(2x^2 + 3x + 1)}$

7. $\dfrac{3x^2 - 2x - 5}{x^3 + x^2}$

8. $\dfrac{3x^2 - x + 1}{x(x + 1)^2}$

9. $\dfrac{x + 1}{3(x - 2)^2}$

10. $\dfrac{3x - 4}{(x - 5)^2}$

11. $\dfrac{8x^2 + 15x + 9}{(x + 1)^3}$

12. $\dfrac{6x^2 - 5x}{(x + 2)^3}$

In Exercises 13–32, find the indefinite integral.

13. $\displaystyle\int \frac{1}{x^2 - 1}\,dx$

14. $\displaystyle\int \frac{9}{x^2 - 9}\,dx$

15. $\displaystyle\int \frac{-2}{x^2 - 16}\,dx$

16. $\displaystyle\int \frac{-4}{x^2 - 4}\,dx$

17. $\displaystyle\int \frac{1}{3x^2 - x}\,dx$

18. $\displaystyle\int \frac{3}{x^2 - 3x}\,dx$

19. $\displaystyle\int \frac{1}{2x^2 + x}\,dx$

20. $\displaystyle\int \frac{5}{x^2 + x - 6}\,dx$

21. $\displaystyle\int \frac{3}{x^2 + x - 2}\,dx$

22. $\displaystyle\int \frac{1}{4x^2 - 9}\,dx$

23. $\displaystyle\int \frac{5 - x}{2x^2 + x - 1}\,dx$

24. $\displaystyle\int \frac{x + 1}{x^2 + 4x + 3}\,dx$

25. $\displaystyle\int \frac{x^2 + 12x + 12}{x^3 - 4x}\,dx$

26. $\displaystyle\int \frac{3x^2 - 7x - 2}{x^3 - x}\,dx$

27. $\displaystyle\int \frac{x + 2}{x^2 - 4x}\,dx$

28. $\displaystyle\int \frac{4x^2 + 2x - 1}{x^3 + x^2}\,dx$

29. $\displaystyle\int \frac{4 - 3x}{(x - 1)^2}\,dx$

30. $\displaystyle\int \frac{x^4}{(x - 1)^3}\,dx$

31. $\displaystyle\int \frac{3x^2 + 3x + 1}{x(x^2 + 2x + 1)}\,dx$

32. $\displaystyle\int \frac{3x}{x^2 - 6x + 9}\,dx$

In Exercises 33–40, evaluate the definite integral.

33. $\displaystyle\int_4^5 \frac{1}{9 - x^2}\,dx$

34. $\displaystyle\int_0^1 \frac{3}{2x^2 + 5x + 2}\,dx$

35. $\displaystyle\int_1^5 \frac{x - 1}{x^2(x + 1)}\,dx$

36. $\displaystyle\int_0^1 \frac{x^2 - x}{x^2 + x + 1}\,dx$

37. $\displaystyle\int_0^1 \frac{x^3}{x^2-2}\,dx$

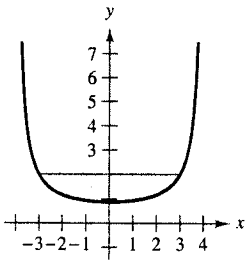
38. $\displaystyle\int_0^1 \frac{x^3-1}{x^2-4}\,dx$

39. $\displaystyle\int_1^2 \frac{x^3-4x^2-3x+3}{x^2-3x}\,dx$

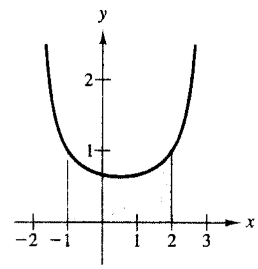
40. $\displaystyle\int_2^4 \frac{x^4-4}{x^2-1}\,dx$

In Exercises 41–44, find the area of the shaded region.

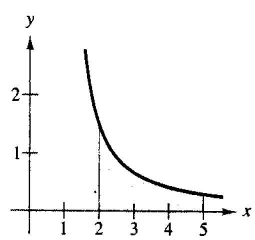
41. $y = \dfrac{14}{16-x^2}$



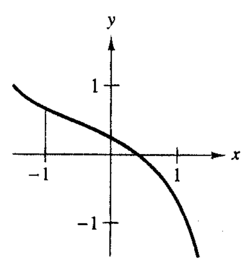
42. $y = \dfrac{-4}{x^2-x-6}$



43. $y = \dfrac{x+1}{x^2-x}$



44. $y = \dfrac{x^2+2x-1}{x^2-4}$



In Exercises 45–48, write the partial fraction decomposition for the rational expression. Check your result algebraically. Then assign a value to the constant a and use a graphing utility to check the result graphically.

45. $\dfrac{1}{a^2-x^2}$

46. $\dfrac{1}{x(x+a)}$

47. $\dfrac{1}{x(a-x)}$

48. $\dfrac{1}{(x+1)(a-x)}$

In Exercises 49–52, use a graphing utility to graph the function. Then find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis by using the integration capabilities of a graphing utility and by integrating by hand using partial fraction decomposition.

49. $y = \dfrac{10}{x(x+10)},\ y=0,\ x=1,\ x=5$

50. $y = \dfrac{-4}{(x+1)(x-4)},\ y=0,\ x=0,\ x=3$

51. $y = \dfrac{2}{x^2-4},\ x=1,\ x=-1,\ y=0$

52. $y = \dfrac{25x}{x^2+x-6},\ x=-2,\ x=0,\ y=0$

53. ***Biology*** A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a capacity of 1000 animals and that the herd will grow according to a logistic growth model. That is, the size y of the herd will follow the equation

$$\int \frac{1}{y(1000-y)}\,dy = \int k\,dt$$

where t is measured in years. Find this logistic curve. (To solve for the constant of integration C and the proportionality constant k, assume $y = 100$ when $t = 0$ and $y = 134$ when $t = 2$.) Use a graphing utility to graph your solution.

54. ***Health: Epidemic*** A single infected individual enters a community of 500 individuals susceptible to the disease. The disease spreads at a rate proportional to the product of the total number infected and the number of susceptible individuals not yet infected. A model for the time it takes for the disease to spread to x individuals is

$$t = 5010\int \frac{1}{(x+1)(500-x)}\,dx$$

where t is the time in hours.

(a) Find the time it takes for 75% of the population to become infected (when $t = 0$, $x = 1$).

(b) Find the number of people infected after 100 hours.

55. ***Marketing*** After test-marketing a new menu item, a fast-food restaurant predicts that sales of the new item will grow according to the model

$$\frac{dS}{dt} = \frac{2t}{(t+4)^2}$$

where t is the time in weeks and S is the sales (in thousands of dollars). Find the sales of the menu item at 10 weeks.

56. ***Biology*** One gram of a bacterial culture is present at time $t = 0$, and 10 grams is the upper limit of the culture's weight. The time required for the culture to grow to y grams is modeled by

$$kt = \int \frac{1}{y(10-y)}\,dy$$

where y is the weight of the culture (in grams) and t is the time in hours.

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$y = \frac{}{1}$

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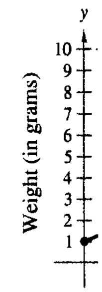
(b) Use the g



57. ***Revenue*** T
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58. ***Medicine***
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59. ***Biology*** A
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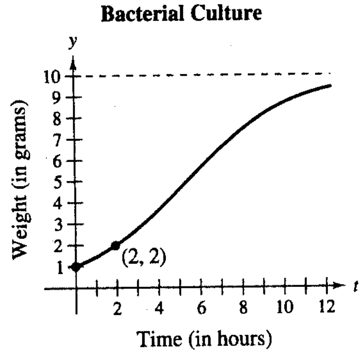
(a) Verify that the weight of the culture at time t is modeled by

$$y = \frac{10}{1 + 9e^{-10kt}}.$$

Use the fact that $y = 1$ when $t = 0$.

(b) Use the graph to determine the constant k.

Bacterial Culture



57. ***Revenue*** The revenue R (in millions of dollars per year) for Symantec Corporation from 1995 through 2003 can be modeled by

$$R = \frac{410t^2 + 28{,}490t + 28{,}080}{-6t^2 + 94t + 100}$$

where $t = 5$ corrresponds to 1995. Find the total revenue from 1995 through 2003. Then find the average revenue during this time period. *(Source: Symantec Corporation)*

58. ***Medicine*** On a college campus, 50 students return from semester break with a contagious flu virus. The virus has a history of spreading at a rate of

$$\frac{dN}{dt} = \frac{100e^{-0.1t}}{(1 + 4e^{-0.1t})^2}$$

where N is the number of students infected after t days.

(a) Find the model giving the number of students infected with the virus in terms of the number of days since returning from semester break.

(b) If nothing is done to stop the virus from spreading, will the virus spread to infect half the student population of 1000 students? Explain your answer.

59. ***Biology*** A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes the population of the species will increase at a rate of

$$\frac{dN}{dt} = \frac{125e^{-0.125t}}{(1 + 9e^{-0.125t})^2}$$

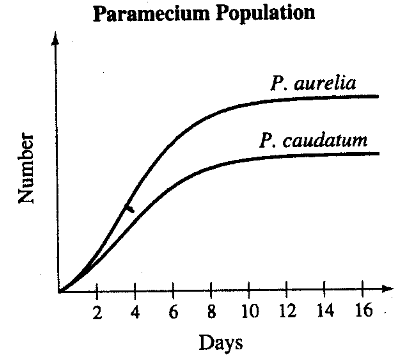
where N is the population and t is the time in months.

(a) Use the fact that $N = 100$ when $t = 0$ to find the population after 2 years.

(b) Find the limiting size of the population as time increases without bound.

60. ***Biology: Population Growth*** The graph shows the logistic growth curves for two species of the single-celled *Paramecium* in a laboratory culture. During which time intervals is the rate of growth of each species increasing? During which time intervals is the rate of growth of each species decreasing? Which species has a higher limiting population under these conditions? *(Source: Adapted from Levine/Miller,* Biology: Discovering Life, *Second Edition)*

Paramecium Population



BUSINESS CAPSULE

Courtesy of Susie Wang/Aqua Dessa

While a math communications major at the University of California at Berkeley, Susie Wang began researching the idea of selling natural skin-care products. She used $10,000 to start her company, Aqua Dessa, and uses word-of-mouth as an advertising tactic. Aqua Dessa products are used and sold at spas and exclusive cosmetics counters throughout the United States.

61. ***Research Project*** Use your school's library, the Internet, or some other reference source to research the opportunity cost of attending graduate school for 2 years to receive a Masters of Business Administration (MBA) degree rather than working for 2 years with a bachelor's degree. Write a short paper describing these costs.