Spring 2019, Math 320: Portfolio Project

Instead of sizable midterm and final exams, for this class you will complete a portfolio thoughout the semester documenting the development of your proof-writing. The portfolio will highlight the growth of your proof-writing skills, the process you employ when doing mathematics, and what you learned from the mistakes you encountered along the way.

In completing your portfolio, the following will be expected.

- The portfolio will be assembled exactly as outlined in this document. There are 4 "sections" outlined below, each consisting of 1 or more "components" that are similarly structured. There will be **10 components total** in the portfolio.

- You must submit some of the components at each of the following periodic deadlines throughout the semester. These can be from any of the 4 sections (i.e. you may "mix-and-match" components for the periodic deadlines however you wish) but they cannot overlap with previously submitted components.

    - Tuesday, March 19th (3 components)
    - Thursday, April 18th (2 components)
    - Thursday, May 2nd (2 components)

    After the last deadline above, you will have completed 7 components, leaving 3 more to be completed before the final submission.

- All components from all sections **must be submitted together** on the day of the final exam (including components submitted at periodic deadlines) **in a binder, report cover, folder, etc.** Each of the **10 components** will be worth 8 points, and meeting each deadline will earn an additional 5 points, for a total of 100 points.

Some additional things to keep in mind.

- It is **imperative** that you **keep all old homework assignments** throughout the semester. Numerous components require submitting/annotating **original** copies of your homework submissions, making them impossible to complete if you do not have the originals.

- Keep the different components in mind as you complete the weekly homework assignments, so they do not sneak up on you at the end of the semester. Note that some components require holding onto extra materials (e.g. your scratchwork for the "Process"section).

- When including an old homework submission, make sure it is clear which part of the page is relevant. It should be **painfully obvious** to the reader.

- This class is an ideal opportunity to learn and practice typesetting mathematics in LaTeX. The software is free and easy to install, and there are many free tutorials available online to help you get started. I am also happy to help you if you run into any issues setting up the software or locating the appropriate commands.

- Do not hesitate to reach out to me with questions as you work on your portfolio! I am happy to help you narrow down which of your homework submissions to use.

The specific sections of the portfolio are outlined in detail below.

Growth. The goal of this section is to illustrate your growth as a proof-writer in this class. For each component of this section, you will locate a proof-based homework problem for which your solution contained at least one error, and correct that error.

In this section, you must submit **at least 3 components**. Each component consists of:

(a) the original copy of a homework submission containing at least one error;

(b) a new writeup containing a correct proof, preserving as much of the content of your original submission as possible (i.e. fix only the parts that are broken); and

(c) a summary (1 paragraph) of the mathematical issues with your proof in part (a) and what steps you took to correct them in part (b).

Assumptions. The goal of this section is to highlight the role assumptions (i.e. hypotheses) play in mathematical statements and their proofs. For each component of this section, you will choose a homework problem wherein you were asked to prove a statement with one or more assumptions, and demonstrate why those assumptions are present.

In this section, you must submit **at least 2 components**. Each component consists of:

(a) the original copy of your homework submission, annotated **in a different color** to indicate where each assumption was used in the proof; and

(b) an explanation (1 paragraph per assumption) of why each assumption must be present for the original statement to be true, including (if possible) a counterexample to the original statement were that assumption to be omitted.

As an example, suppose you chose the following problem.

"If $f : A \to B$ is onto and A is finite, then B is finite."

This problem has 2 assumptions: (i) f is onto, and (ii) A is finite. For the first assumption, one possible counterexample would be $A = \{1, 2, 3\}$, $B = \mathbb{Z}$, and $f : A \to B$ given by $f(a) = a$ for each $a \in A$. Your explanation should state why this example violates assumption (i), why it does **not** violate assumption (ii), and why the conclusion "B is finite" is violated.

Process. The goal of this section is to demonstrate the importance of "scratchwork" when writing proofs. For each component of this section, you will choose a homework problem where you used scratchwork to assemble a proof in a non-sequential manner. Examples of this include (i) working backwards from the conclusion, (ii) identifying a key step in the middle before finishing the rest of the proof before/after that key step, or (iii) starting with several concrete examples to see how an argument might be constructed.

In this section, you must submit **at least 2 components**. Each component consists of:

(a) the original copy of your homework submission;

(b) the scratchwork you produced when completing this problem, annotated **in a different color** to indicate which parts were helpful in writing the final proof; and

(c) a summary (1 paragraph) of the process you used to solve this problem that connects the scratchwork and the final proof.

Conjectures. The goal of this section is to practice "doing new mathematics" by formulating mathematical questions and subsequently answering them. For each component of this section, you will pick a topic you found interesting from class, formulate a conjecture (something that you think might be true that is not presented in class or in the text) and try to prove it. Of the components you include in your portfolio, **at least one conjecture must be true**.

In this section, you must submit **at least 1 component**. Each component consists of:

(a) a formal statement of your conjecture;

(b) the scratchwork that led to your conclusion on the conjecture;

(c) a proof of the conjecture, or a counterexample (whichever is appropriate); and

(d) a summary (1 paragraph) of your explorations and the process you used to determine whether your conjecture was true.