Spring 2019, Math 320: Preliminary Problem Set 4
Due: Thursday, February 21st, 2019
Introduction To Rings

Preliminary problems. These problems should be completed before discussion on Thursday.

(P1) Listed below are the axioms for $(R, +, \cdot)$ to be a field. Fill in the blanks.

- For every $a, b, c \in R$,
  - ______________________________ (addition is *closed*)
  - ______________________________ (addition is *associative*)
  - ______________________________ (addition is *commutative*)
  - ______________________________ (multiplication is *closed*)
  - ______________________________ (multiplication is *associative*)
  - ______________________________ (multiplication is *commutative*)
  - ______________________________ (the *distributive* property)
- There exists elements
  - $0 \in R$ (the *additive identity*) such that ____________________ for all $a \in R$
  - $1 \in R$ (the *multiplicative identity*) such that ____________________ for all $a \in R$
- For each $a \in R$,
  - there is an element $b \in R$ (the *additive inverse* of a) with $a + b =$ _________ $= 0$
  - if $a \neq$ ___, there exists $b \in R$ (the *multiplicative inverse* of a) with $ab = ba =$ ___

(P2) Write "(I)" next to each item above that is needed to ensure $(F, +)$ is an integral domain. Hint: this requires adding an additional axiom at the end.

(P3) Write "(R)" next to each axiom item that is needed to ensure $(F, +, \cdot)$ is a ring.