

**Spring 2019, Math 320: Preliminary Problem Set 7**  
**Due: Thursday, March 14th, 2019**  
**Isomorphisms and Homomorphisms**

**Preliminary problems.** These problems should be completed before discussion on Thursday.

(P1) Suppose  $R$  and  $S$  are rings, and  $\phi : R \rightarrow S$  be any function. Write down the requirements for  $\phi$  to be an isomorphism.

(P2) Indicate with an (H) which requirements above are needed for  $\phi$  to be a homomorphism.

(P3) Let

$$R_1 = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\} \quad \text{and} \quad R_2 = \mathbb{C},$$

and consider the **isomorphism**  $\varphi : R_1 \rightarrow R_2$  defined by

$$\varphi \left( \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \right) = a + bi$$

for all  $a, b \in \mathbb{R}$  (you may assume  $\varphi$  is indeed an isomorphism). Compute

$$\varphi \left( \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ -6 & 5 \end{pmatrix} \right)$$

without doing any matrix multiplication.