Spring 2019, Math 596: Problem Set 4
Due: Tuesday, February 26th, 2019
Polytopes

Discussion problems. The problems below should be worked on in class.

(D1) *Warmup.* The goal of this problem is to practice drawing "familiar" polytopes.

(a) Compare your drawings from the preliminary problems.

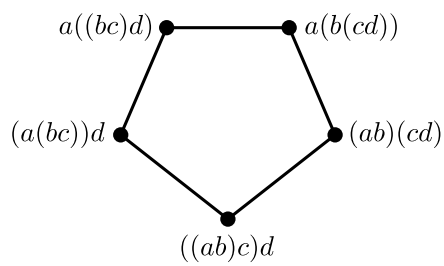
(b) Draw the 3-dimensional simplex, cube, and octohedron. Experiment with different perspectives to find one with maximal clarity. Be sure each member of your group draws at least one!

(c) Draw the 1-skeleton (i.e. just the vertices and edges) of the 3-dimensional simplex, cube, and octohedron in such a way that no edges cross.

(d) Draw the 1-skeleton of the 4-dimensional octohedron. Note: it is provably impossible to do this without edges crossing!

(D2) *Associahedra.* The goal of this problem is to build and draw a new 3-dimensional polytope.

(a) The *associahedron* A_n is a polytope whose vertices are in bijection with the ways of associating parenthesis when multiplying n elements. The polytope A_4 (dimension 2) is depicted below. What is the "rule" for when two vertices are connected by an edge?



(b) List all associations of $abcde$, e.g. $a(b((cd)e))$ or $(ab)((cd)e)$. There are 14 total.

(c) Draw a graph whose vertices are the 14 expressions you found above, and where an edge is drawn between any two vertices that differ by moving parenthesis exactly one.

(d) Find a way to draw the above graph so that no edges cross.

(e) Using the above graph as a starting place, draw the associahedron A_5. Try to make the facets as close to regular polygons (equal edge lengths) as possible. Suggestions:

- Take your time and be methodical in your drawing. Patience is greatly rewarded.
- It may take several revisions to find a "satisfying" drawing, so don't be afraid to experiment and try some different things!
- There is no single "correct" answer here. Unlike most "common" shapes like the cube or octohedron, many different drawings are possible.